Skeleton Based As-Rigid-As-Possible Volume Modeling

Shaoting Zhang, Andrew Nealen and Dimitris Metaxas

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As-rigid-as-possible (ARAP) shape modeling is a popular technique to obtain natural deformations. There have been many excellent methods. However, we do not want to break the manifoldness of ARAP surface modeling or sacrificing the speed. We do it by leveraging the skeleton information.

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Introduction

- As-rigid-as-possible (ARAP) shape modeling is a popular technique to obtain natural deformations. There have been many excellent methods.
- We are interested in the volume preservation.
- VGL is a good approach. However, we do not want to break the manifoldness of ARAP surface modeling or sacrificing the speed.

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Given original coordinates $V$, the connectivity, and $m$ control points, the reconstructed object $V'$ can be obtained by minimizing:

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L \\
l_c
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When rotations are large, the deformation may not be natural.
Iterate two steps to recover rotations:

- Step 1: Initial guess from solving naive LSE.
- Step 2: Find optimal rotations, then update the linear system (edge length preserving).

Robustness, simplicity, efficiency. However, there is no volume preserving constraint.

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Skeleton and volume constraints

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Use both the skeleton and edge length constraint to roughly preserve the volume, without breaking the manifoldness of ARAP or increasing the computation complexity.
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\[
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L & 0 \\
I_c & 0 \\
L_s & 0
\end{bmatrix}
\begin{bmatrix}
V' \\
V_s'
\end{bmatrix}
= 
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\delta \\
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One-way coupling property.
Mesh editing framework

- Manually define the skeleton.

[Image of mesh editing interface with options for defining and editing the skeleton]
Mesh editing framework

- Manually define the skeleton.

Evenly generate skeleton points, and connect them with surface vertices automatically.

Manually select anchor points (bottom) and control points (top).

Interactively deform the shape.
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Experimental settings

- The C++ implementation was run on a Intel Core2 Quad 2.40GHz CPU with 8G RAM.
- We compare the linear LSE, ARAP surface modeling and our method.
- We tested on the cactus model (620 vertices, 1,236 polygons) and the horse model (2,482 vertices, 4,960 polygons).
- The relative root mean square errors of edge lengths and volume magnitudes are reported.
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<table>
<thead>
<tr>
<th>Model</th>
<th>RRMS-E</th>
<th>RE-V</th>
<th>Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>0.126</td>
<td>0.453</td>
<td>0.017</td>
</tr>
<tr>
<td>(c)</td>
<td>0.074</td>
<td>0.131</td>
<td>0.024</td>
</tr>
<tr>
<td>(d)</td>
<td>0.075</td>
<td>0.056</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Results

Model | RRMS-E | RE-V | Times
-----|-------|------|------
(b)  | 0.068 | 0.356| 0.117|
(c)  | 0.040 | 0.125| 0.121|
We proposed an approach to approximately preserve the volume without breaking the manifoldness of traditional ARAP or increasing the computational complexity.

Our method is easy-to-implement and may be useful to systems relying on ARAP techniques.

Limitations: Skeleton generation; complex skeletons; self intersection.
Thanks for listening.