Due Jan 17 | Homework Set 1 (Sections 1.1) | Name:_____

1. Use induction to prove that $1 + r + \cdots + r^{n-1} = \frac{r^n - 1}{r-1}$ for $n \geq 1$ where $r \neq 1$ is some variable.

Suppose r is some variable which does not equal 1. Consider the basis step where $n=1$. Then the right hand side is $\frac{r-1}{r-1}$ which is equal to the left hand side of 1. Now, suppose k is some integer greater than 0, and $1+r+\cdots+r^{k-1} = \frac{r^k-1}{r-1}$.

Consider now: $1+r+\ldots+r^k = \frac{r^k-1}{r-1} + r^k = \frac{r^k - 1 + r^k(r-1)}{r-1} = \frac{r^{k+1}-1}{r-1}$.

Hence, by induction, $1+r+\ldots+r^{n-1} = \frac{r^n-1}{r-1}$.

2. Use induction to prove that $n < 2^n$ for all $n \geq 0$.

For $n=0$, the given inequality is $0<2^0$. For $n=1$, the inequality is $1<2^1$. For $n=2$, the inequality is $2<2^2$. Thus, for $0 \leq n \leq 2$, $n<2^n$. This is the basis step. Suppose $j<2^j$ for any $0 \leq j < k$. ~~Consider~~ Then $k = 1+(k-1) \leq 1+2^{k-1} \leq 2^{k-1}+2^{k-1} = 2^k$ for any $k \geq 2$. Thus, by induction, $n<2^n$ for any $n \geq 0$.

3. Find the quotient and remainder when 1612 is divided by 74. Be sure to write your answer in the form: $a = bq + r$.

Let $a = 1612$ and $b = 74$. Apply the Division Algorithm to get that $1612 = 74(21) + 58$.

4. Use the Division Algorithm to prove that the cube of any integer has to be exactly one of these forms: $9k$ or $9k + 1$ or $9k + 8$ for some integer k.

Let a be some integer. Then by the Division Algorithm, $a = 3q$, $3q+1$, or $3q+2$ for some integer q. Consider a^3. If $a=3q$, then $a^3 = (3q)^3 = 27q^3 = 9(3q^3)$. Set $k=3q^3$, and $a^3 = 9k$ for integer k. If $a = 3q+1$, then $a^3 = (3q+1)^3 = 27q^3 + 27q^2 + 9q + 1 = 9(3q^3+3q^2+q)+1$. Set $k = 3q^3+3q^2+q$, and $a^3 = 9k+1$ for integer k. If $a = 3q+2$, then $a^3 = (3q+2)^3 = 27q^3 + 54q^2 + 36q + 8 = 9(3q^3+6q^2+4q)+8$. Set $k = 3q^3+6q^2+4q$; then $a^3 = 9k+8$ for an integer k. Thus, we get the desired result.

5. a) Read Appendix A, and answer the following questions. What are the methods used to write proofs? Have we used any of these methods in class? If so, which one(s)?

b) Find an article written in a mathematics journal (Notices of the American Mathematical Society is one such journal). Answer the following questions:

Author:
Article Title:
Journal Title:
Vol. and Issue Number:
Page Numbers: