Math 1120 Section 5 **Calculus** **Test 4**

April 25, 2005 **Name** ________________________________

The total number of points available is 165. Throughout this test, **show your work.**

1. (20 points) Consider the function $f(x) = x^2 e^{x/3}$.

   (a) Find all the critical points of f.

   (b) Find the sign chart for f'.

   (c) Find the intervals over which f is increasing.

2. (20 points) Consider the function $f(x) = (\ln x)/x$. Note that f is defined only when $x > 0$.

   (a) Use the quotient rule to find $f'(x)$.

   (b) Use the quotient rule to find $f''(x)$.

   (c) Find the sign chart for $f''(x)$.

   (d) Find the intervals over which f is concave upwards.

3. (15 points) Solve each of the following equations.

   (a) $5e^{.08t} = 10$

   (b) $\dfrac{50}{1+2e^{2x}} = 10$

   (c) $2^{2x} - 5 \cdot 2^x + 6 = 0$ (Hint $2^{2x} = (2^x)^2$, so the given equation is a quadratic. Try factoring the left side.)

4. (8 points) How much money should be deposited in a bank account paying an interest rate of 6% per year compounded monthly so that at the end of 3 years the accumulated amount is $20,000.

5. (8 points) At what rate of interest does it take six years of compounding monthly for an account to double in value?

6. (8 points) How long does it take a 5% investment, compounded continuously, to triple in value?

7. (20 points) Consider the function $f(x) = e^{x^3-3x^2}$.

   (a) Compute $f'(x)$.

   (b) Find the critical points of f.

   (c) Find the relative max and min of f.

   (d) What is the maximum value of f over the interval $[-2, 4]$?

8. (16 points) Let $f(x) = \ln(4 + x^2)$.

   (a) Find $f'(x)$.

   (b) Find $f'(2)$.

   (c) Find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

9. (10 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 110e^{-0.01t}$$

where $m(t)$ is measured in grams.

(a) Find the mass at time $t = 0$.

(b) How much of the mass remains after 20 years?

(c) At what rate is the mass declining after 10 years?

10. (20 points) A study finds that the average student taking advanced shorthand progresses according to the function

$$Q(t) = 120(1 - e^{-0.05t}) + 60, \quad (0 \le t \le 20),$$

where $Q(t)$ measures the number of words (per minute) of dictation that the student can take in machine shorthand after t weeks in the twenty-week course. Sketch the graph of Q and answer the following questions:

(a) What is the beginning shorthand speed for the average student?

(b) What shorthand speed does the average student attain halfway through the course?

(c) How many words per minute can the average student take at the end of the course.

(d) What is the rate of change of the speed after exactly 5 weeks in the course?

11. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the difference between the object's temperature and that of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $70°F$, A and k are constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 20 minutes, the temperature of the coffee is $186.6°F$. What is the value of k?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

(d) Find the rate at which the object is cooling after $t = 20$ minutes.