April 4, 2005 **Name** ______________________________

The total number of points available is 157. Throughout this test, **show your work.**

1. (24 points) Consider the function $f(x) = \frac{2x+9}{6x+3}$. For this function there are two important intervals: $(-\infty, A)$ and (A, ∞) where the function is not defined at A.

   (a) What is A?

   (b) Compute $f'(x)$.

   (c) Construct the sign chart of $f'(x)$.

   (d) Find the intervals over which $f(x)$ is decreasing.

   (e) Does f have any inflection points?

   (f) Find an interval over which f is concave upwards.

   (g) Find all vertical asymptotes.

   (h) Find all horizontal asymptotes.

2. (15 points) Consider the function $f(x) = 3x + 9x^{-1}$. For this function there are four important intervals: $(-\infty, A]$, $[A, B)$,$(B, C]$, and $[C, \infty)$ where A, and C are the critical numbers and the function is not defined at B.

   (a) Find A, B, and C.

   (b) For each of the following intervals, tell whether $f(x)$ is increasing or decreasing over $(-\infty, A]$,$[A, B)$, $(B, C]$, and $[C, \infty)$

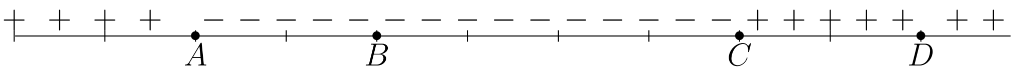
   (c) Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether $f(x)$ is concave up or concave down over each of the intervals $(-\infty, B)$ and (B, ∞)

3. (12 points) Find the critical points of each function.

   (a) $f(x) = (x^2 - 4)^2(2x - 3)^3$

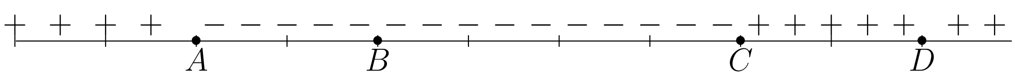
   (b) $g(x) = (x^2 - 9)^{2/3}$

4. (16 points) Given below is a sign chart for the derivative $f'(x)$ of a function.

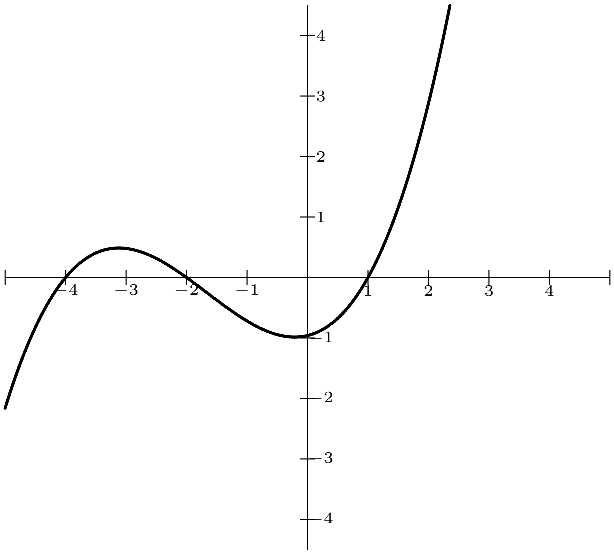


(a) For each of the stationary points A, B, C and D tell whether $f(x)$ has a relative maximum, relative minimum, or neither at the point.

(b) Suppose $f(x)$ is a polynomial function with critical points A, B, C and D. Sketch a function on the coordinate system below that could have a derivative whose sigh chart is the one given.



5. (10 points) Consider the cubic polynomial $p(x)$ whose graph is given. On the same coordinate grid, sketch the derivative function $p'(x)$.



6. (10 points) Suppose the function $f(x)$ has been differentiated twice to get $f''(x) = (x-2)(x+1)(x+3)$. Find the intervals over which $f(x)$ is concave upward.

7. (25 points) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $f(x) = 11 - x^2$. For example two of the vertices of the rectangle could be $(-3, 0)$ and $(3, 0)$, both on the x-axis. Then the other two vertices would be $(-3, f(-3)) = (-3, 2)$ and $(3, f(3)) = (3, 11 - (3)^2) = (3, 2)$. In this case the area of the rectangle is $A = 6 \cdot 2 = 12$.

   (a) Now suppose we use $x = 2$ to get a vertex. Then one vertex is $(2, 0)$. What are the other three vertices?

   (b) What is the area of the rectangle determined by this choice $x = 2$?

   (c) How does the area depend on x. In other words, if R is the rectangle determined by x, (and $-x, f(x), f(-x)$), what is the area $A(x)$ of R?

   (d) What choices of x give rise to rectangles? In other words, what is the domain of the function in part 3.

   (e) What are the dimensions of such a rectangle with the greatest possible area?

8. (25 points) Let $g(x) = 2x^3 - 36x^2 + 120x + 4$.

   (a) Find the critical points of g.

   (b) Find the intervals over which g is increasing.

   (c) Find the intervals over which g is concave upward.

   (d) Find the locations of local maxima and minima for g.

   (e) What is the maximum value of g over the interval $[0, 10]$?

9. (20 points) Amber Airlines runs chartered flights to Costa Rica. They expect 300 passengers and they charge each passenger \$200. However if more than 300 persons sign up for the flight, they agree to charge \$0.25 less per ticket for each extra person. For example, if 302 passengers sign up, the airline charges each of the 302 passengers \$199.50.

   (a) Find the revenue function $R(x)$ in terms of the number of new passengers x. In other words, let $x + 300$ represent the number of passengers, where $x > 0$.

   (b) How many passengers result in the maximum revenue?

   (c) What is that maximum revenue?