March 16, 2005 **Name** _____

The total number of points available is 140. Throughout this test, **show your work.**

1. (12 points) Let $f(x) = \sqrt{x^3 - x + 3}$.

   (a) Compute $f'(x)$

   (b) What is $f'(2)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} 3x - x^3 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x^{2/3} & \text{if } x > 1 \end{cases}$$

   (a) Is f continuous at $x = 1$?

   (b) What is the slope of the line tangent to the graph of f at the point $(8, 8)$?

   (c) Find $f'(-3)$

3. (12 points) If a ball is thrown vertically upward from the roof of 112 foot building with a velocity of 48 ft/sec, its height after t seconds is $s(t) = 112 + 48t - 16t^2$.

   (a) What is the height the ball at time $t = 0$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) What is the maximum height the ball reaches?

   (d) What is the velocity of the ball when it hits the ground (height 0)?

4. (7 points) The cost of producing x units of stuffed alligator toys is $C(x) = 0.003x^2 + 6x + 6000$. Find the marginal cost at the production level of 1000 units.

5. (30 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = g(x)/f(x)$. Compute $L'(2)$.

(b) Let $U(x) = f \circ f(x)$. Compute $U'(1)$.

(c) Let $K(x) = f(x^2) \cdot g(x)$. Compute $K'(1)$

(d) Let $V(x) = g \circ g(x)$. Compute $V'(5)$.

(e) Let $W(x) = f(x - g(x))^2$. Compute $W'(4)$.

(f) Let $Z(x) = g(x + f(x))$. Compute $Z'(3)$.

6. (25 points) Compute the following derivatives.

   (a) Let $f(x) = 1/x^2 + \sqrt{x^3}$. Find $\frac{d}{dx}f(x)$.

   (b) Let $g(x) = \sqrt{x^3 - x^2}$. What is $g'(x)$?

   (c) Find $\frac{d}{dx}((4x+1)^2 \cdot (2x^3 - 1))$.

   (d) Find $\frac{d}{dx}\frac{2x^3+1}{x-2}$.

   (e) Find $\frac{d}{dt}(t^3 + 1/t)^4$.

7. (30 points) Let $f(x) = \sqrt{x^2 - 3}$. The chain rule can be applied to find that $f'(x) = \frac{1}{2}(x^2 - 3)^{-1/2} \cdot 2x = x/\sqrt{x^2 - 3}$. Use the limit definition of derivative to verify this fact. That is, use the four step procedure you learned in the first part of the course.

8. (12 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{2x - 5}$ and let $[a, b] = [3, 15]$. Finally, suppose $M = 4$. Find the number c whose existence is guaranteed by IVT.