

Find the domain and the intervals of concavity of $g(x) = -\sqrt{4-x^2}$. First note that g is defined only when $4-x^2 \geq 0$ and this turns out to be $-2 \leq x \leq 2$. To find g' , rewrite g in fractional exponential form, $g(x) = -(4-x^2)^{1/2}$. Now,

$$\begin{aligned}g'(x) &= -\frac{1}{2}(4-x^2)^{-1/2}(-2x) \\ &= x(4-x^2)^{-1/2}.\end{aligned}$$

Therefore we can find g'' by the product rule.

$$\begin{aligned}g''(x) &= 1(4-x^2)^{-1/2} + \left(-\frac{1}{2}(4-x^2)^{-3/2}\right)(-2x) \cdot x \\ &= (4-x^2)^{-1/2} + x^2(4-x^2)^{-3/2} \\ &= (4-x^2)^{-1/2} \left(1 + x^2(4-x^2)^{-1}\right) \\ &= \frac{1}{(4-x^2)^{1/2}} \left(\frac{4-x^2}{4-x^2} + \frac{x^2}{(4-x^2)}\right) \\ &= \frac{1}{(4-x^2)^{1/2}} \left(\frac{4}{(4-x^2)}\right) \\ &= \frac{4}{(4-x^2)^{3/2}}\end{aligned}$$

There are two (equivalent) ways to interpret $r^{3/2}$. One is $\sqrt{r^3}$ and the other is $(\sqrt{r})^3$ and both these result in a positive answer when r is itself positive. Of course, since the $4-x^2$ term is in the denominator, we must eliminate both 2 and -2 . For all the numbers $x \in (-2, 2)$, $g''(x) > 0$. IE, g is concave upwards on $(-2, 2)$. Note that the graph of g is just the bottom half of the circle $x^2 + y^2 = 4$.