Introducing Some Economics Terminology

This essay is intended to be read before the second test. The purpose is to familiarize you with some terminology economists use and to provide some examples of the use of calculus in economics. Five terms used consistently in a calculus for business course are *supply, demand, cost, revenue*, and *profit.* It is important to know these terms and also how they relate to one another. For purposes of discussion, we assume that a given company manufactures a single item. Let x denote the number of these made weekly.

The demand for the item is a function $p = p(x)$ which is the price the company can charge at a level x of production. Since the demand is a measure of the consumers' interest in the item, large x is associated with lower prices and vise-versa. The supply, on the other hand, is a measure of the size of production, and this depends on the manufacturer's willingness to produce the item. Thus supply is an increasing function of price: the higher the price, the more items will be produced.

Revenue, denoted $R(x)$, is simply a function that relates the number of items produced with the company's income from the sale of these items. If $p(x)$ is the price per item when x items are produced, then $R(x) = xp(x)$ is the revenue. Be sure you understand why this is true. For example, if $p(x) = 100 - x$, then $R(x) = x(100 - x) = 100x - x^2$. Next let $C(x)$ denote the total cost function (aka cumulative cost function). So $C(x)$ denotes the cost of manufacturing x items. Note that the cost of producing just one item, say the 100^{th} one is $C(100) - C(99)$. In this course the cost function $C(x)$ is usually quadratic with positive linear term and negative quadratic term. In the example $C(x) = 8000 + 200x - 0.02x^2$, we can see that the constant term (the fixed cost) does not change as the number of items produced changes. This number might come from the purchase of the land for the plant or from the building of the plant. The linear term could be the cost of materials, labor, and power to run the assembly line. Next the profit function $P(x)$ is simply the difference between the revenue and the cost:

$$P(x) = R(x) - C(x).$$

Associated with each of these functions three functions is its *marginal*, ie. its derivative with respect to x. Hence marginal cost is $C'(x)$, marginal revenue is $R'(x)$, marginal profit is $P'(x)$. In addition, we sometimes work with the *average* function, denoted $\overline{C}(x), \overline{P}(x)$, and $\overline{R}(x)$. These are defined in just the way you anticipate: $\overline{C}(x) = C(x)/x, \overline{P}(x) = P(x)/x$, and $\overline{R}(x) = R(x)/x$. When we want to emphasize that we are talking about the cost of producing a single item we sometimes use the word 'incremental' because it suggests that this number is a difference of two C values. Thus the cost, revenue, and profit for the production level of, for example, the 100^{th} item would be $C(100)-C(99), R(100)-R(99)$, and $P(100)-P(99)$. These three values can be approximated remarkably well by the values $C'(100), R'(100)$, and $P'(100)$. The reason for this interesting approximation is that, for example,

$$P'(100) = \lim_{h \to 0} \frac{P(100+h) - P(100)}{h}.$$

But notice that when $h = 1$ (a small value compared with $x = 100$), we get

$$P'(100) \approx \frac{P(101) - P(100)}{1}.$$

Here is a sample problem that illustrates some of the ideas in this essay. The fixed cost of producing x widgets per week is \$10000, and the production cost is \$50 per unit. But by buying in bulk, the company can save $\$0.003x^2$ on x widgets. The relationship between price and demand for widgets is given by $p = f(x) = -0.04x + 300, \quad 0 \leq x \leq 7000$, where p is the price in dollars.

1. What is the cost function?

   Solution: $C(x) = 10000 + 50x - 0.003x^2, \quad 0 \leq x \leq 1000.$

2. Find the average cost function $\overline{C}(x)$.

   Solution: $\overline{C}(x) = 10000/x + 50 - 0.003x.$

3. Find the (incremental) cost of producing the 500$^{\text{th}}$ widget.

   Solution: $C(500) - C(499) = 10000 + 50 \cdot 500 - 0.003 \cdot 500^2 - (10000 + 50 \cdot 499 - 0.003 \cdot 499^2) = 50 - 0.003(999) = 47.003.$

4. Find the marginal cost function $C'(x)$.

   Solution: $C'(x) = 50 - 0.006x.$

5. What is $C'(500)$? Compare this number with the number you found in part 3.

   Solution: $C'(500) = 50 - 0.006(500) = 50 - 3 = 47.$

6. Find the marginal average cost function $\overline{C}'(x)$.

   Solution: $\overline{C}'(x) = -10000/x^2 - 0.003.$

7. Find the revenue function $R(x)$.

   Solution: $R(x) = xp = xf(x) = x(-0.04x + 300) = -.04x^2 + 300x.$

8. Find the marginal revenue function $R'(x)$.

   Solution: $R'(x) = -0.08x + 300.$

9. What is the maximum revenue?

   Solution: The revenue is maximized at $x = 3750$, so the largest it gets is $R(3750) = 562500$.

10. Find the profit function $P(x)$.

    Solution: $P(x) = R(x) - C(x) = -.04x^2 + 300x - (10000 + 50x - 0.003x^2) = 250x - 0.037x^2 - 10000$.

11. Find the marginal profit function $P'(x)$.

    Solution: $P'(x) = 250 - 0.074x$.

12. Find a value of x where the profit function $P(x)$ has a horizontal tangent line.

    Solution: Solve $P'(x) = 0$ for x to get $x = 250 \div 0.074 \approx 3378$.

13. What is the maximum profit?

    Solution: The profit is maximized at $x = 3378$, so the largest it gets is $P(3378) = 260287$.