The twelvefold way

Putting k balls into n boxes.

Domain (size k)	Target (size n)	All May receive 0	1-1 (injective) Each receives ≤ 1	Onto (surjective) Each receives ≥ 1
dist.	dist.	n^k	$(n)_k$	$n!S(k,n)$
id.	dist.	$\left(\!\binom{n}{k}\!\right)$	$\binom{n}{k}$	$\left(\!\binom{n}{k-n}\!\right)$
dist.	id.	$S(k,1)+\cdots+S(k,n)$	$\delta_{k\leq n}$	$S(k,n)$
id.	id.	$P(k,1)+\cdots+P(k,n)$	$\delta_{k\leq n}$	$P(k,n)$

Explanation:

$(n)_k = n\cdot(n-1)\cdots(n-k+1)$ is a falling factorial.

$\binom{n}{k}$ is a binomial coefficient (the number of k-element subsets of an n-element set).

$\left(\!\binom{n}{k}\!\right) = \binom{n+k-1}{k}$ is the number of k-element multisets chosen from an n-element set.

$S(k,n)$ is the number of ways to partition a k-element set into n classes or parts. (A Stirling number of the second kind).

$P(k,n)$ is the number of partitions of the integer k into n parts.

$\delta_{k\leq n}$ is 1 if $k \leq n$ and zero otherwise.

Note: When $k \leq n$, the sum $S(k,1)+\cdots+S(k,n)$ is known as the Bell number B_k. (Obviously $S(k,k+1) = S(k,k+2) = \cdots = S(k,n) = 0$).