Assignment 9

Oral questions

1. Let O be the center of the circle of inversion, P' the inverse of P and Q' the inverse of Q. Assume that O, P, and Q form a triangle. Show that $OPQ_{\triangle}$ is similar to $OQ'P'_{\triangle}$. Use this result to show that inversion preserves the cross-ratio: if A, B, P, and Q are four points distinct from the center O of the circle of inversion and A', B', P', and Q' are their inverses then $(AB, PQ) = (A'B', P'Q')$.

2. Prove that inversion preserves the angle of two circles or lines, in the special case when you have two lines that intersect in a point that is different from the center of the base circle. (Recall that the inverses of these lines are circles that contain the center of the base circle.)

Questions to be answered in writing

1. Prove that inversion about the circle given by $x^2 + y^2 = 1$ takes the point $(x, y) \neq 0$ into the point $(x/(x^2 + y^2), y/(x^2 + y^2))$.

2. Prove that inversion preserves the angle of two circles, using the statements on our handout, in the special case when the center of the base circle and the centers of the two other circles are collinear. Assume the center of the base circle is 0 and its radius is 1. Assume the two circles to be inverted have their centers O_1 and O_2 on the real line, at c_1 and c_2 respectively, and that they have radius r_1 and r_2 respectively. Assume the point P is an intersection of these circles. Using the law of cosines, express the cosine of $\angle O_1PO_2$ in terms of c_1, c_2, r_1, r_2. Let O_1', O_2' and P' the image of O_1, O_2 and P under the inversion. Using the formulas on our handout, show that $\angle O_1'P'O_2'$ has the same cosine.