# Assignment 8

## Oral questions

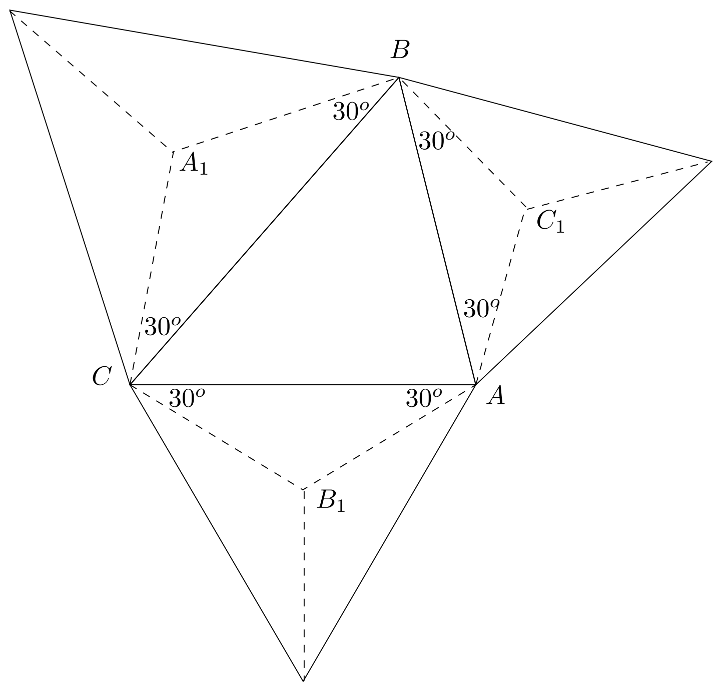
1. 4.7/13 (Nagle point)

2. For a triangle $\triangle ABC$ let $A'$, $B'$, and $C'$, respectively, be the points where the incircle is tangent to the sides $BC$, $AC$, and $AB$, respectively. Prove that the lines $AA'$, $BB'$ and $CC'$ are concurrent. (The common intersection is the *Gergonne point*.)

## Questions to be answered in writing

1. Use Ceva's theorem to prove that the orthocenter exists.

2. Prove Napoleon's theorem: Given an arbitrary triangle $ABC_\triangle$, the centers of the equilateral triangles exterior to $ABC_\triangle$ form an equilateral triangle. (Illustration and hints on next page.)




*Hints:* Represent the points $A, B, C, A_1, B_1, C_1$ with complex numbers $a, b, c, a_1, b_1, c_1$. Observe that multiplying with

$$\rho := \frac{1}{\sqrt{3}} \left(\cos(30^o) + i \cdot \sin(30^o)\right)$$

rotates the vector $\overrightarrow{BA} = a - b$ into $\overrightarrow{BC_1} = c_1 - b$. Use this observation to express $c_1$ in terms of $a, b$ and $\rho$. Express then $a_1$ and $c_1$ similarly in terms of $a, b, c$ and $\rho$. Show that $c_1 - a_1$ is obtained by multiplying $b_1 - a_1$ with

$$\frac{\rho}{1-\rho} = \frac{2\rho - 1}{\rho} = \frac{\rho - 1}{2\rho - 1}.$$

It is probably easier to do so if you find the quadratic equation whose roots are $\rho$ and its conjugate. Finally show that

$$\frac{\rho}{1-\rho} = \cos(60^o) + i \cdot \sin(60^o)$$

meaning that $\overrightarrow{A_1C_1}$ is obtained from $\overrightarrow{A_1B_1}$ by a $60^o$ rotation.