Assignment 6

Oral questions

1. Prove the following converse of the inscribed angle theorem ("Star Trek Lemma"): given a circle centered at O, and three points A, B, and C such that

   (i) B and C are on the circle,
   (ii) The angle $\angle BAC$ is the half of $\angle BOC$,

   the angle A is also on the circle. (I ask you to work out only the case when $\angle BAC$ is acute and O lies in its interior, keeping in mind that there are also other cases, see the first written question. It might help if you consider, how $\angle BAC$ changes when you move the point A on a line containing O, towards O or away from it.)

2. Prove that a quadrilateral is cyclic if and only if the sum of two of its opposite angles is 180^o. Explain which implication is related to the Star Trek Lemma, and which to its converse.

3. Let a, b, and c be the sides of a triangle, and A its area. Prove that the excircle at side a has radius $2A/(-a+b+c)$.

Questions to be answered in writing

1. 4.4/15

2. Use Heron's formula to find the radius of the inscribed circle of the triangle, whose side lengths are 3, 3, and 5.