Sample Test II.

The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides the questions listed below, any question that is similar to a homework question from the first three chapters may occur on the test.

1. Find $\frac{d}{dt}(\mathbf{c}_1(t)\cdot\mathbf{c}_2(t))$ and $\frac{d}{dt}(\mathbf{c}_1(t)\times\mathbf{c}_2(t))$ for $\mathbf{c}_1(t)=(e^t,t,t^2)$ and $\mathbf{c}_2(t)=(1,t,\sin(t))$.
2. A particle of mass is moving at constant speed s in a circular path of radius 2 around a body with mass 10. Use Newton's law to find s.
3. Find the arc-length of the curve $(1,\sqrt{2}t,t^2)$ for the interval $0\le t\le 2$.
4. Verify that the curve $(\sin(t),\cos(t))$ is a flow line of the velocity field $F(x,y)=(-y,x)$.
5. Find the divergence and curl of the vector field $\mathbf{V}(x,y,z)=xy\mathbf{i}+e^z\mathbf{j}+xz\mathbf{k}$.
6. Find the scalar curl of the gradient of the function $f(x,y)=\cos(xy)$.
7. Find $\int_1^2\int_0^3 \frac{e^x}{1+y^2}\,dy\,dx$.
8. Sketch the region D that represents the region of integration for $\int_0^2\int_{-\sqrt{9-y^2}}^0 f(x,y)\,dx\,dy$.
9. Assume you want to evaluate $\int_0^1\int_0^x f(x,y)\,dy\,dx$ by changing the order of integration. How do the limits of integration change?
10. Describe the unit ball as an elementary region.
11. Find $\int_0^1\int_0^y\int_0^x z\,dz\,dx\,dy$.
12. Integrate $\cos(x^2+y^2)$ over the disk $x^2+y^2\le 4$. Use integration by substitution, switch to polar coordinates.
13. Suppose you want to find $\int\int_D f(x,y)\,dx\,dy$ by using the map $T:D^*\to D$ given by $x(u,v)=u^2\cdot v$ and $y(u,v)=1+v$. Find the Jacobian of this substitution.
14. Find the average value of the function $f(x,y)=x^2-y$ over the triangle whose vertices are $(0,0)$, $(1,0)$ and $(1,1)$.
15. Find the center of mass of the region bounded by $y=x$ and $y=x^2$ if the density is $x+y$. (Center of mass formula will be provided for such a question.)
16. Find a parametrization for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

17. Evaluate the path integral $\int_{\mathbf{c}} f(x,y,z)\ ds$ where $f(x,y,z) = x+y+z$ and $\mathbf{c}(t) = (\sin t, \cos t, 3)$.

18. The force field is $\mathbf{F} = (x, y, 1)$. Compute the work done moving along the parabola $(t, t^2, 0)$ from $t = 0$ to $t = 3$.

19. Evaluate $\int_{\mathbf{c}} x\ dy - y\ dx$ where $\mathbf{c}(t)$ is any parametrizaton of the unit circle. Explain why this question can only be answered up to sign.

20. Find $\int_{\mathbf{c}} \nabla f \cdot\ ds$ where $f(x,y) = x^2 + y$ and $\mathbf{c} : [0,1] \to \mathbb{R}^3$ is a piecewise C^1 path starting at $(1,3)$ and ending at $(4,5)$.

21. Assume that the path $\mathbf{c} : [a,b] \to \mathbb{R}^2$ has the property that $\mathbf{c}'(t)$ is nowhere zero. Explain how computing $\int_{\mathbf{c}} \mathbf{F}\ ds = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\ dt$ may be reduced to computing the path integral of a scalar function.

Good luck. Gábor Hetyei