Sample Test I.

The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides the questions listed below, any question that is similar to a homework question from the first three chapters may occur on the test.

1. Find the parametric equation of the plane given by the points $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, 1)$. Then find also its equation by finding its normal vector.
2. Find the orthogonal projection of the vector $(1, 2, 3)$ onto the vector $(2, 1, 1)$.
3. *State and prove the Cauchy-Schwartz inequality in* $\mathbb{R}^n$..
4. *State and prove the triangle inequality in* $\mathbb{R}^n$.
5. Find the distance between the point $(1, 1, 2)$ and the plane given by $x + y + z = 1$.
6. Find the volume of the paralelepiped spanned by $\mathbf{i}$, $\mathbf{i} - 2\mathbf{j}$, and $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.
7. Find the product AB of $A = \begin{pmatrix} 1 & 0 & 1 \\ -3 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 5 & 0 \\ 0 & -1 \end{pmatrix}$.
8. Find the spherical and cylindrical coordinates of the point, whose Cartesian coordinates are $(6, 0, 8)$. (I will provide formulas for spherical coordinates, if needed.)
9. Describe the following level surfaces of $f(x, y, z) = x^2+y^2-z^2$: $f(x, y, z) = 2$ and $f(x, y, z) = -1$.
10. Find the equation of the tangent plane of the function $f(x, y) = x^2 - \cos(y)$ at $(1, 0)$. Use it to approximate $f(1.01, 0)$.
11. Is the function $f(x, y) = x^{1/3}y^{1/3}$ differentiable at $(0, 0)$? Does it have partial derivatives?
12. Find the gradient of $f(x, y) = x^2 \cos(y)$.
13. Fill in the expressions "differentiable", "continuous partials", and "partials exists" into the following diagram (the symbol "$\Rightarrow$" stands for "implies"):

$$\underline{\hspace{4cm}} \Rightarrow \underline{\hspace{4cm}} \Rightarrow \underline{\hspace{4cm}}.$$

14. Find the equation of the tangent line of the curve $(\cos(t), \sin(t), t)$ at $t = \pi/2$.
15. Use the chain rule to find the derivative of $f \circ g$ at $(1, 0)$ where $f : \mathbb{R}^3 \to \mathbb{R}^2$ is given by $f(u, v, w) = (\sin^2(u) - w, e^w)$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $g(x, y) = (e^y, x - y, x)$.

16. Suppose you are at the point $(2, 3)$. Find the direction in which the function $f(x, y) = x^2y$ increases the fastest. How does this direction relate to the tangent line of the level curve $f(x, y) = 12$ at $(2, 3)$?

17. Find $\partial^2 f/\partial x\partial y$ of $f(x, y) = e^x/y$ at $(2, 1)$. How does this compare to $\partial^2 f/\partial y\partial x$ at the same point?

18. Find the equation of the plane tangent to the surface $x^2 - 2y^2 + xz = 4$ at the point $(1, 0, 3)$.

19. Find the directional derivative of $f(x, y) = \ln(x^2 + y^2 + 1)$ at $(1, 0)$ in the direction of $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

Good luck. Gábor Hetyei