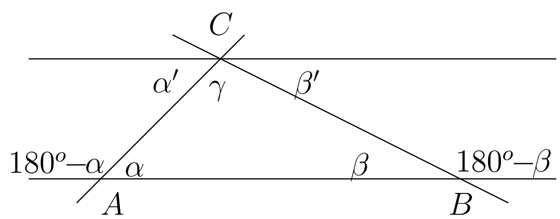
Assignment 2

Oral questions

1. List all primitive Pythagorean triples (a, b, c) that satisfy $b = 20$. (Here b is one of the legs.) How many primitive Pythagorean triples satisfy $b = 30$?

2. Given a line ℓ and a point P on it, define the following relation for on the points of $\ell \setminus \{P\}$: $A \sim B$ if P is not between A and B, that is, $A * P * B$ is false. Prove that this relation is an equivalence relation.

3. Complete the following proof of the theorem stating that the sum of the angles of a triangle ABC is 180^o. We draw parallel line to AB through C and use the notation introduced in the picture.



Applying Euclid's fifth postulate to the line AB and the angles $180^o - \alpha$ and α' yields $180^o - \alpha + \alpha' \geq 180^o$. As a consequence we must have $\alpha' \geq \alpha$. Similarly, applying Euclid's fifth postulate to the line BC and the angles $180^o - \beta$ and β' yields $180^o - \beta + \beta' \geq 180^o$, and so $\beta' \geq \beta$. Hence we obtain

$$\alpha + \beta + \gamma \leq \alpha' + \beta' + \gamma \leq 180^o.$$

Use Euclid's fifth postulate directly in two more situations to show that $\alpha + \beta + \gamma$ is also greater than equal to 180^o.

Questions to be answered in writing

1. Explain how Thales' theorem is a special case of the Star Trek Lemma. Prove Thales' theorem. Prove the Star Trek Lemma in the case when the angle $\angle BOC$ is acute and O is on the line segment AB.

2. Assume that the distance of the points O_1 and O_2 is d. Draw a circle of radius r_1 around O_1 and a circle of radius r_2 around O_2. Express, in terms of equations and inequalities for r_1, r_2 and d, necessary and sufficient conditions for the two circles to have 0, 1 or 2 points in common. (You do not have to prove your claims, but you have to consider all possibilities, including one circle containing the other one.)