Sample Test I.

The real test will have less questions and you will have about 80 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work.

1. How many 5-letter words can you form using the letters a, b, c, d, e such that at least one letter is repeated?

   Solution: I provide a solution to this one only since it applies the *subtraction principle* (not in the book). Sometimes it is easier to count in a superset what is not in your solution set and subtract the number obtained from the size of the superset. In this example the total number of 5-letter words (superset) is 5^5, the number of 5-letter words with no letter repeated is $5!$. Your answer is $5^5 - 5!$.

2. How many ways are there to select a committee of 3 women and 2 men out of 10 women and 5 men?

3. You roll a dice three times. How many outcomes are there?

4. How many ways are there to paint 20 identical eggs using 4 colors if every egg is painted using only one color?

5. How many ways are there to line up 2 red, 3 blue, and 4 green balls?

6. What is the coefficient of xy^6z in $(2x + y^2 - z)^5$?

7. How many ways are there to distribute 6 different books among 10 children if no child may get more than one book?

8. How many ways are there to select 5 coins from an unlimited supply of nickels, dimes and quarters?

9. Write up the first 6 rows of Pascal's triangle.

10. Prove the binomial identity
$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}.$$

11. Prove the binomial identity
$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$

12. Simplify the expression
$$\sum_{k=0}^{n} \binom{n}{k} kx^k.$$

13. Simplify the expressions

$$1 + 1/3 + 1/9 + 1/27 + \cdots + 1/3^n \quad \text{and} \quad \sum_{n=0}^{\infty} \frac{1}{2^n}.$$

14. Find the coefficient of x^{10} in $(x^2 + x^3 + x^4 + x^5 + \cdots)^3$. What is the coefficient of x^r?

15. Use generating functions to find the number of ways to distribute r jelly beans among 6 children if the number of jelly beans received by each child is a multiple of 3 and each child receives at least one jelly bean.

16. List all partitions of the integer 5.

17. Let e_k denote the numbers of k's in a partition of r. What equation must the numbers $e_1, e_2, \ldots$ and e_r satisfy? Use this equation to find the generating function for the number of partitions of a number.

18. Find the generating function for the number of ways to express the number r as a sum of distinct integers.

19. Find the exponential generating function for the number of ways to place r distinct people into 5 different rooms such that no room is left empty and the number of people in each room is a multiple of 3.

Good luck.   Gábor Hetyei