Assignment 11

Oral questions

1. A hyperbolic circle centered at C of radius r is the set of all points A satisfying $d(A, C) = r$. Prove that a hyperbolic circle corresponds to a circle in the Poincaré disk model, although its center may not be equal to its Euclidean center. (Hint: prove the statement in the case when $C = P$ first, where P is the center of the Poincaré disk. Use then the fact that reflection about the perpendicular bisector of PC takes a hyperbolic circle centered at P into a hyperbolic circle centered at C, and that this reflection corresponds to an inversion about a circle.)

2. Assume $a, b, c \in \mathbb{R}$ satisfy $a^2 + bc = 1$, and let $T : \mathbb{C} \to \mathbb{C}$ be given by
$$T(z) = \frac{a\overline{z} + b}{c\overline{z} - a}.$$
Show that $T(T(z)) = z$ for all z. (All reflections of the Poincaré upper half plane model are represented by such a function.)

3. All hyperbolic rotations fixing the point i in the Poincaré upper half plane model are fractional linear transformations $z \mapsto \frac{az+b}{cz+d}$ sending i into i. Using this fact, and assuming that we have scaled our coefficients to satisfy $ad - bc = 1$, show that
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$
for some angle θ.

Question to be answered in writing

1. Find the Poincaré distance between the points $P = 3 + i$ and $Q = (6 + \sqrt{2})/2 + \sqrt{2}/2 \cdot i$.