Assignment 4

Oral questions

1. Represent points in the plane using their Cartesian coordinates. Explain how Pythagoras' theorem implies the *distance formula* stating that the distance between the points $A = (a_1, a_2)$ and $B = (b_1, b_2)$ is $|AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$. Use this formula to verify that the midpoint of the line segment AB is $M = \left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}\right)$. (Don't forget to check that M is on the line AB.)

2. Use the midpoint formula shown in the previous exercise to prove the existence of the centroid as follows. Consider the triangle $ABC_\triangle$ where $A = (a_1, a_2)$, $B = (b_1, b_2)$ and $C = (c_1, c_2)$. Prove that the point $G = (\frac{a_1+b_1+c_1}{3}, \frac{a_2+b_2+c_2}{3})$ belongs to all three medians. Thus not only all three medians intersect in a common point but we have a formula to express this point in terms of the coordinates of the vertices. Introducing A_1 for the midpoint of the line segment BC, prove that the distance $|A_1G|$ is the half of the distance $|GA|$.

3. Consider a point P outside a circle Γ . Draw a tangent from P to the circle and denote the point where the tangent meets Γ by T. Prove that the power of P with respect to Γ is $|PT|^2$ in two ways: note that this is already implicit in our notes and directly, by comparing $|PT|^2$ to a $|PQ| \cdot |PQ'|$, using similar triangles.

Question to be answered in writing

1. Consider a triangle $ABC_\triangle$ and choose a Cartesian coordinate system that has its origin at the circumcenter O of $ABC_\triangle$. Using the notation $\underline{a} := \overrightarrow{OA}$, $\underline{b} := \overrightarrow{OB}$, $\underline{c} := \overrightarrow{OC}$, and inner products, show that the point H defined by $\overrightarrow{OH} = \underline{a}+\underline{b}+\underline{c}$ has the following property: the line AH is orthogonal to the line BC (and, similarly $BH \perp AC$ and $CH \perp AB$). Thus the orthocenter must exist. Using the formulas from the oral exercises, explain how this formula also implies the theorem about the Euler line (not only its existence, but also the stated distance proportions!)