Study Guide for the Midterm Exam

1 Definitions and axioms to remember

1. Axioms: Euclid's postulates (I may ask you also about Birkhoff's and Hilbert's postulates, but if I do so, I will provide a copy of those), postulates from the section "Transformation geometry".
2. Definitions: triangular and square numbers, rays, line segments, distance function, isometry, congruence, similarity, power of a point, rational, irrational, algebraic, transcendent numbers, Fermat primes, and the following triangle centers: centroid, orthocenter, incenter, circumcenter, Fermat point. You should also be able to use inner products and complex numbers.

2 Statements you should remember with their proof

1. From our textbook: Equivalence of Playfair's postulate to Euclid's fifth postulate and theorems about triangle congruences. Pythagoras' theorem (+converse). The Star Trek Lemma (+converse, from hw), special cases such as Thales' theorem, application to cyclic quadrilaterals. Lemma 5.1, graph of a linear equation, existence of a power of a point, law of cosines, extended law of sines. Existence of: centroid, incenter, circumcenter, orthocenter, Euler line. Constructibility of the sum, product, quotient, and square root of given distances (also from lecture). Explanation how the question of constructing a regular polygon with n sides may be reduced to the question when n is the power of an odd prime (see also lecture).
2. From lecture and handouts: Parallel projection theorem (as much as we covered), Fermat point, Ceva's theorem, existence of isogonal conjugates. I expect you to be able to prove the existence of such points as the Nagel point or the Lemoine point using Ceva's theorem, but I will provide the definition of such points. (I also may define a triangle center for you that you never heard of before and ask you to use Ceva's theorem to prove its existence.)
3. From homework: Gregory's formula for π, sum of the interior angles of a triangle, formula for the radius of the excircle.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

3 Statements you should know (without proof)

1. From our textbook: Heron's formula, the description of Feuerbach's circle, compass equivalence theorem, the existence of the constructions listed in section 6.2, construction of a regular pentagon (you should know at least one method), description of distances that are not constructible (involving more than square root), description of those primes p for which a regular pentagon with p^k sides is constructible. Impossibility to square the circle, trisect most angles, and double the cube.
2. From lecture: description of Pythagorean triplets.

Of course you have to remember the standard trigonometic identities.

4 What to expect

The exam will be *closed book*. You will have 80 minutes. Some questions may ask you to state and prove a theorem from the list I gave, others may be exercises similar to your homework assignments. There may be questions about examples, whether they have certain properties. You may have to describe a geometric construction. You do not have to bring a compass and ruler, bit if you describe your construction in words, no detail should be missing.