

# Study Guide for the Final

## 1 Definitions, axioms and models to remember

1. Axioms: Euclid's postulates, Playfair's postulate, hyperbolic axiom.
2. Definitions: transversal, defect of a triangle, horoparallel and hyperparallel lines, pencil of lines, fractional linear transformations (also conjugate ones!), hypercycles, horocycles, hyperbolic trigonometric functions.
3. Poincaré disk model and upper half plane model: points, lines, angle, and distance in these models.

## 2 Statements you should remember with their proof

1. From our textbook: Alternate interior angle theorem, weak exterior angle theorem, unique midpoint of a line segment, unique bisector of an angle, Saccheri-Legendre theorem, additivity of the defect, zero defect or rectangle exists iff the geometry is Euclidean (Theorem 7.6). (AAA) congruence in hyperbolic geometry, angle of parallelism depends only on the distance between the point and the line, existence uniqueness of a limiting parallel ray (Theorem 8.2), circumference of the hyperbolic circle in the Poincaré disk model.
2. From lecture and handouts: inversion takes circles and lines into circles and line, exact formulas, with proof. Inversion preserves angles and cross ratio. Theorem 14.1 and Lemmas 14.1-14.3 on horoparallel lines. No three points on a hypercycle are collinear. Formulas for a hyperbolic right triangle (15.4) and (15.5), hyperbolic Pythagorean theorem.
3. From homework: Hyperbolic circles are also Euclidean circles in the Poincaré disk model. Fractional linear transformations preserve angles and cross-ratio, description of fractional linear transformations preserving the upper half plane, description of rotations around  $i$  in the Poincaré upper half plane model. Complex cross ratio is real for points on a circle or line.

If a proof was covered in several ways you may choose your favorite one. You may also invent your own proof.

## 3 Statements you should know (without proof)

1. From our textbook: (SAA) congruence, statements on Saccheri and Lambert quadrilaterals in Section 7.6, hyperparallel lines have a common perpendicular, statements on singly and doubly asymptotic triangles in sections 7.9 and 7.10, Theorems 8.7 and 8.8 on limiting parallel rays, classification of parallel lines, Lobachevskii's formula for the angle of parallelism (Theorem 10.10), defect is proportional to the area. I might ask you to apply the hyperbolic law of sines or cosines (Theorems 15.2 and 15.3) but then I will provide the formulas.

## 4 What to expect

The exam will be *closed book*. The above guide is meant to help with the mandatory part. For the optional part prepare as if it was another midterm. The mandatory part will be as long as the midterm, the optional part will have only about 5 questions.