Study Guide for the Midterm Exam

1   Definitions to remember

1. continuous functions (Definition 17.1 and Theorem 17.2), uniform continuity (Definition 19.1), limits of functions (Definitions 20.1, 20.3, and Theorem 20.6; only left, right, and two-sided limits, essentially Corollaries 20.7 and 20.8), variations on the definition of the limit ("Discussion" 20.9), derivatives (Definition 28.1). *You have to remember the proofs of the theorems mentioned above.*

2. From the lecture: right and left derivatives.

2   Statements you should remember with their proof

1. From the book: Theorems 17.3, 17.4, and 17.5 (continuity is preserved by some operations), 18.1 (continuous function on a closed bounded interval reaches its maximum, *plus* counterexamples to generalizations), 19.2 (on a closed bounded interval, continuous $\Rightarrow$ uniformly continuous, *plus* counterexamples to generalizations), 19.4 (uniformly continuous function takes Cauchy sequence into Cauchy sequence), 20.4 and 20.5 (limits and certain operations), 20.10 (two-sided limit exists iff. both one-sided limits exist, and they are equal), 28.2 (differentiable $\Rightarrow$ continuous, *plus* counterexample to opposite implication), 28.3 (derivatives and basic operations).

2. From your homework assignments: all mandatory exercises (oral or written) that were listed in the first six assignments.

3   Statements you should know (without proof)

1. From the book: Theorems 16.2, 16.3, and 16.5 (decimal expansion), 18.2 (intermediate value), Corollary 18.3, Theorems 18.4, 18.5, 18.6 (one-to-one continuous functions on an interval are the strictly monotone functions, and their inverse is also continuous), 19.5 (uniformly continuous on (a, b) $\Leftrightarrow$ it may be continuously extended to $[a, b]$), 19.6 (derivative bounded $\Rightarrow$ uniformly continuous), Theorem 28.4 (chain rule).

2. From the lecture: The bonus questions on merging sequences with the same limit, and $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$.

4   What to expect

The exam will be *closed book*. You will have 80 minutes. Some questions may ask you to state and prove a theorem from the list above, others may be exercises similar to your homework assignments. There may be questions about examples, whether they have certain properties. You may have to give a counterexample to a false statement. You may have to calculate the limit or the derivative of a function, in some cases, using the definitions only, in others you will be allowed to use all rules learned. You may also have to convert a rational number to a repeating decimal or vice versa.