Sample Test 2

The actual test will only have about 5 questions

1. Using the definition of the derivative as the limit of the difference quotient, find the derivative of $f(x) = x^3 - 4x^2$ at $x = 3$.

2. Using the definition of the derivative as the limit of the difference quotient, find the derivative of $f(x) = \sqrt{x}$.

3. Using the definition of the derivative as the limit of the difference quotient, find the derivative of $f(x) = 1/x$.

4. Use the basic rules for derivatives, to find the equation of the tangent line of $f(x) = x^3 - 40 \cdot \sqrt{x}$ at $x = 4$.

5. Using the product rule, find the derivative of $(x^2 + 3x) \cdot (x^3 + 1)$.

6. Using the quotient rule, find the derivative of $\frac{x^2-3}{x+1}$.

7. Using the chain rule, find the derivative of $y = \sqrt{x^2 - 2x + 5}$.

8. The price of a product is given by $p = 400 - x^2$ where x is the number of items sold. Find the revenue and the marginal revenue for $x = 10$.

9. For a certain product, the number x of items sold is the following function of the price p: $x = \sqrt{50 - p}$. What is the elasticity of the demand? Is the demand at price level $p = 1$ elastic, inelastic or unitary? Use the formula $E(p) = -\dfrac{p \cdot f'(p)}{f(p)}$.

10. The distance s (in feet) covered by a car after t seconds is given by $s(t) = -t^3 + 55t^2 + 21t$. Find the formula expressing the velocity and acceleration of the car after t seconds (in feet/sec^2).

11. The distance s (in feet) covered by a car after t seconds is given by $s(t) = -t^3 + 7.5t^2 - 18t$. Find the formula expressing the velocity car after t seconds (in feet/sec^2). When will the car turn around for the first time?

Solutions:

1. $f(3+h) = h^3 + 5h^2 + 3h - 9$ and $f(3) = -9$, so
$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{h^3 + 5h^2 + 3h}{h} = \lim_{h\to 0} h^2 + 5h + 3 = 3.$$

2.
$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}. \end{aligned}$$

3.
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}.$$

4. The derivative is $f'(x) = 3x^2 - \frac{20}{\sqrt{x}}$, so the slope of the tangent line is $f'(4) = 38$. If $x = 4$ then $f(4) = -16$. The equation of the tangent line is $y + 16 = 38(x-4)$, that is, $y = 38x - 168$.

5.
$$\begin{aligned} ((x^2+3x)\cdot(x^3+1))' &= (x^2+3x)'\cdot(x^3+1) + (x^2+3x)\cdot(x^3+1)' \\ &= (2x+3)\cdot(x^3+1) + (x^2+3x)\cdot 3x^2 = 5x^4 + 12x^3 + 2x + 3. \end{aligned}$$

6.
$$\begin{aligned} \left(\frac{x^2-3}{x+1}\right)' &= \frac{(x^2-3)'\cdot(x+1) - (x^2-3)\cdot(x+1)'}{(x+1)^2} = \frac{2x\cdot(x+1) - (x^2-3)\cdot 1}{(x+1)^2} \\ &= \frac{x^2+2x+3}{(x+1)^2}. \end{aligned}$$

7. The outer function is $y = \sqrt{u}$, the inner function is $u = x^2 - 2x + 5$. The derivatives are $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$ and $\frac{du}{dx} = (2x - 2)$. The derivative is

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2 \cdot \sqrt{x^2 - 2x + 5}} \cdot (2x - 2) = \frac{x - 1}{\sqrt{x^2 - 2x + 5}}.$$

8. The revenue is given by $R(x) = x \cdot (400 - x^2) = 400x - x^3$. Substituting $x = 10$ gives a revenue of $R(10) = 3000$. The derivative of $R(x)$ is $400 - 3x^2$ and so the marginal revenue is $R'(10) = 100$.

9. For $f(p) = \sqrt{50 - p}$ the chain rule gives

$$f'(p) = \frac{1}{2\sqrt{50 - p}} \cdot (-1) = \frac{-1}{2\sqrt{50 - p}}$$

Substituting into the formula for $E(p)$ gives

$$E(p) = -\frac{p \cdot \frac{-1}{2\sqrt{50-p}}}{\sqrt{50 - p}} = \frac{p}{100 - 2p}$$

Hence $E(1) = 1/98 < 1$ and the demand is inelastic at $p = 1$.

10. We need to find the first two derivatives. The first derivative is $s'(t) = -3t^2 + 110t + 21$, this is the velocity. The second derivative is $s''(t) = -6t + 110$, this is the acceleration.

11. We need to find the first derivative, which is $s'(t) = -3t^2 + 15t - 18 = -3(t - 2)(t - 3)$. The smaller zero of this polynomial is at $t = 2$.