Sample Test II.

The real test will have less questions and you will have about 75 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides looking at the sample questions below, also review all homework exercises, as questions similar to them might occur on the test.

1. State the 4-color theorem and explain how it applies to map coloring. In particular, does it mean that every map can be colored using four colors?

2. What is the chromatic number of a wheel graph with n vertices?

3. Suppose the maximum degree in a graph is 10. According to Brooks' theorem, which is the only graph with this property, whose vertices require 11 colors to be properly colored?

4. The maximum degree of a vertex in a graph G is 5. What can you say about the minimum number of colors needed to properly color its edges?

5. What is the chromatic polynomial of a path of length n? Prove your formula.

6. Prove that the chromatic polynomial $P_k(G)$ of a graph G satisfies the recurrence

$$P_k(G) = P_k(G - e) - P_k(G/e).$$

   Here e is an edge of G, and $G - e$, respectively G/e are the graphs obtained by deleting, respectively, contracting the edge e. Explain, how the above recurrence may be used to prove that $P_k(G)$ is a polynomial. (What is the basis for the induction?)

7. Find the chromatic polynomial of the graph shown in Figure 1.

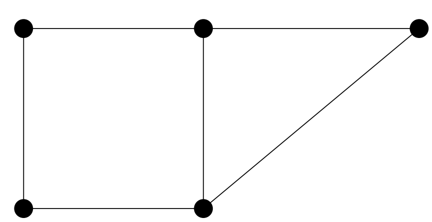

Figure 1: Graph to exercise finding the chromatic polynomial

8. Outline the proof of the fact that every planar graph may be properly colored using at most 5 colors.

9. Give at least two different definitions of a tree (page 94, Theorem 1).

10. Explain why each tree must have at least one leaf. (Also define leafs.)

11. State the number of edges in a tree on n vertices and prove your formula using induction and the statement in the previous question.
12. Explain how and why the formula on the number of edges in a tree follows from Euler's formula.
13. Find a breadth-first search and a depth-first search spanning tree in the edge graph of the cube.
14. Explain how depth-first search may be used to find your way out of a maze.
15. Explain how search trees may be used to solve the missionaries and cannibals puzzle, and present a solution to this puzzle.

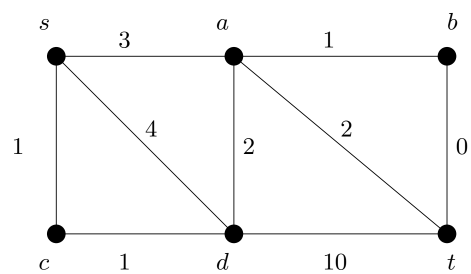

Figure 2: Graph to exercise Dijkstra's algorithm

16. Explain why Dijkstra's algorithm finds the shortest path. Illustrate the algorithm by finding the shortest path between s and t in Figure 2.
17. Prove that Kruskal's algorithm finds a minimum weight spanning tree.
18. Using Kruskal's algorithm find a minimum weight spanning tree for the graph shown in Figure 3. List the edges of the tree in the order you included them during the algorithm.

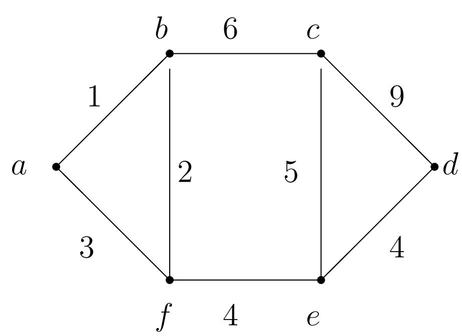

Figure 3: Graph to exercise Kruskal's algorithm

Good luck. Gábor Hetyei