

Sample Test II.

The real test may have less questions and you will have about 80 minutes to answer them. The usage of books or notes, or communicating with other students will not be allowed. You will have to give the simplest possible answer and show all your work. Besides this sample test, review also your homework problems!

1. Find the chromatic polynomial of the graph shown in Figure 1.

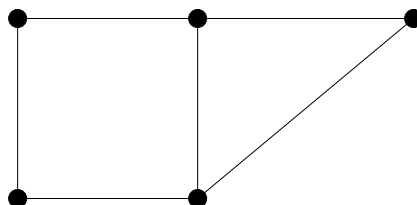


Figure 1: Graph to exercise finding the chromatic polynomial

2. Outline the proof of the fact that every planar graph may be properly colored using at most 5 colors.
3. Give at least two different definitions of a tree (page 94, Theorem 1).
4. Explain why each tree must have at least one leaf. (Also define leaves.)
5. State the number of edges in a tree on n vertices and prove your formula using induction and the statement in the previous question.
6. Explain how and why the formula on the number of edges in a tree follows from Euler's formula.
7. Find a breadth-first search and a depth-first search spanning tree in the edge graph of the cube.
8. Explain how depth-first search may be used to find your way out of a maze.
9. Explain how search trees may be used to solve the missionaries and cannibals puzzle, and present a solution to this puzzle.
10. Explain why Dijkstra's algorithm finds the shortest path. Illustrate the algorithm by finding the shortest path between s and t in Figure 2.

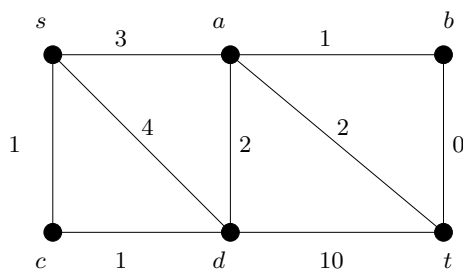


Figure 2: Graph to exercise Dijkstra's algorithm

11. Prove that Kruskal's algorithm finds a minimum weight spanning tree.
12. Using Kruskal's algorithm find a minimum weight spanning tree for the graph shown in Figure 3. List the edges of the tree in the order you included them during the algorithm.

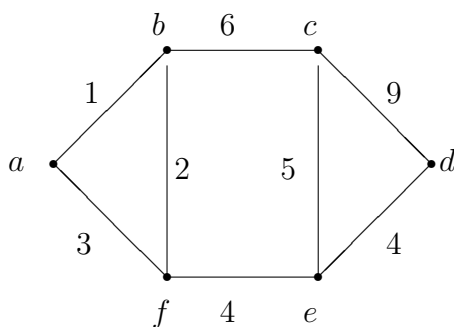
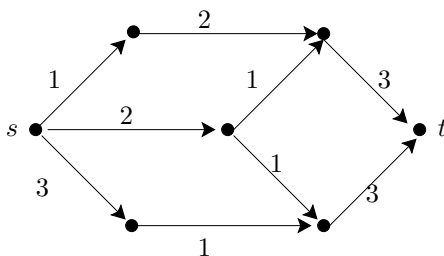


Figure 3: Graph to exercise Kruskal's algorithm

Figure 4: Network with source s and sink t

13. Find a minimum cut and a maximum flow for the network shown in Figure 4. **Show all your work!**
14. Give an example of a maximal flow that is not maximum. Explain how finding an augmenting path in the network of residual capacities can help correct mistakes.

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Test II

Student ID:

15. State the Ford-Fulkerson theorem for network flows. Explain how the network flow algorithm may be used to prove it for integer capacities.
16. State Menger's theorems and explain how network flows may be used to prove Menger's theorem for pairwise edge disjoint paths. (See your notes and "messenger problems" in the book).

Good luck.

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