Correction to the Transportation Problem Example
given in Section 4.5 in Tucker's "Applied Combinatorics", 5th Ed.

Follow what is written in the book up to and including formula (5) at the end of section 4.5. Now go back to step II. B. but use the u_i's and v_j's given in formula (5) and disregard (6) which we don't know where it came from.

edge $(1,1)$: $c_{11} = 4 > v_1 - u_1 = \$11 - \$10 = \$1$ increase of $\$3$
edge $(1,3)$: $c_{13} = 0 > v_3 - u_1 = \$6 - \$10 = -\$4$ increase of $\$4$
edge $(2,3)$: $c_{23} = 0 < v_3 - u_2 = \$6 - \$5 = \$1$ decrease of $\$1$
edge $(3,1)$: $c_{31} = 5 = v_1 - u_3 = \$11 - \$6 = \$5$ no change

Thus the solution may be improved by adding the edge $(2,3)$ to the current spanning tree, whose edge set is $\{(1,2),(2,1),(2,2),(3,2),(3,3)\}$. Adding the edge $(2,3)$ to the tree creates the circuit $\{(2,2),(2,3),(3,2),(3,3)\}$. Since nothing is transported along the edge $(2,2)$, we can not change the solution of the transportation problem. But we can change the associated spanning tree to $\{(1,2),(2,1),(2,3),(3,2),(3,3)\}$ and this will improve our set of selling prices. Note that both $\{(1,2),(2,1),(2,2),(3,2),(3,3)\}$ and $\{(1,2),(2,1),(2,3),(3,2),(3,3)\}$ have an edge with 0 flow in it, present only to make sure our solution is associated to a spanning tree. We changed this "superfluous" edge. Our solution is now represented by the following table.

	1		2		Dummy	
1		4		2		0
		40				
2	60	6		7		0
				0		
3		5		6		0
		10		20		

Now we work out a new set of prices, starting with an arbitrary price $u_1 = \$10$.

warehouse 1: $u_1 = \$10$ store 1: $v_1 = \$12$
warehouse 2: $u_2 = \$6$ store 2: $v_2 = \$12$
warehouse 3: $u_3 = \$6$ dummy: $v_3 = \$6$

Again we look for a better solution:

edge $(1,1)$: $c_{11} = 4 > v_1 - u_1 = \$12 - \$10 = \$2$ increase of $\$2$
edge $(1,3)$: $c_{13} = 0 > v_3 - u_1 = \$6 - \$10 = -\$4$ increase of $\$4$
edge $(2,2)$: $c_{22} = 7 > v_2 - u_2 = \$12 - \$6 = \$6$ increase of $\$1$
edge $(3,1)$: $c_{31} = 5 < v_1 - u_3 = \$12 - \$6 = \$6$ decrease of $\$1$

The solution may be improved by adding the edge $(3,1)$ to the spanning tree $\{(1,2),(2,1),(2,3),(3,2),(3,3)\}$. This move creates the circuit $\{(2,1),(3,1),(3,3),(2,3)\}$. The lowest even edge has 20 on it, this is the amount by which we can change the flow in the edges. The new solution is shown in the following table.

	1		2		Dummy	
1		4		2		0
			40			
2	6		7			0
	40				20	
3	5		6			0
	20		10			

Note that our flow decreased to zero in the edge $(3,3)$ and we remove this edge to have a spanning tree again. Our new spanning tree is $\{(1,2),(2,1),(2,3),(3,1),(3,2)\}$. We use this tree to work out a new set of prices, starting with an arbitrary price $u_1 = \$10$.

$$
\begin{array}{llll}
\text{warehouse 1:} & u_1 = \$10 & \text{store 1:} & v_1 = \$11 \\
\text{warehouse 2:} & u_2 = \$5 & \text{store 2:} & v_2 = \$12 \\
\text{warehouse 3:} & u_3 = \$6 & \text{dummy:} & v_3 = \$5
\end{array}
$$

Again we look for a better solution:

edge $(1,1)$: $\quad c_{11} = 4 > v_1 - u_1 = \$11 - \$10 = \$1$ increase of $\$3$
edge $(1,3)$: $\quad c_{13} = 0 > v_3 - u_1 = \$5 - \$10 = -\$5$ increase of $\$5$
edge $(2,2)$: $\quad c_{22} = 7 = v_2 - u_2 = \$12 - \$5 = \$7$ no change
edge $(3,3)$: $\quad c_{33} = 0 > v_3 - u_3 = \$5 - \$6 = -\$1$ increase of $\$1$

Now we really have an optimal solution. The cost of this solution is

$$40 \times \$2 + 40 \times \$6 + 20 \times \$0 + 20 \times \$5 + 10 \times \$6 = \$480,$$

$\$20$ less expensive than the one in the book.