

Study Guide for the Midterm Exam

1 Definitions to remember

1. From your notes: equivalence relation, partial order, total order, well-ordered set, dense order.
2. From Landau's book: Definitions 1-39 except for Definitions 14, 23, 35, and 38.

When a Definition is stated in combination with a Theorem, you do not have to remember the proof (unless stated otherwise below) but you have to remember what statement had to be shown to make the Definition meaningful.

3. From your homework assignments: the equivalence relation on ordered pairs of positive integers that we used to construct the set of all integers, their addition, subtraction, and multiplication.

2 Theorems you should remember with their proof

1. From Landau's book: Theorem 15 (transitivity of ordering on \mathbb{P}); Theorem 19 (compatibility of ordering with addition on \mathbb{P}); Theorem 30 (Distributive Law on \mathbb{P}); Theorems 37, 38, 39, and 40 (relation to construct \mathbb{Q}^+ is an equivalence relation, and you may simplify by the same factor); Theorems 53, 54, and 55 (density properties of $\mathbb{Q}^+ \cup \{0, \infty\}$); Theorems 56, 58, and 59 (addition in \mathbb{Q}^+ is well-defined, associative and commutative); Theorems 68, 69, 70, 71 (multiplication in \mathbb{Q}^+ is well-defined, associative, commutative, and it is distributive with respect to the addition); Theorem 111 (\mathbb{P} may be considered as a subset of \mathbb{Q}^+), Theorems 114 and 115 (division and the Archimedean property for \mathbb{Q}^+); Theorem 129 (sum of two cuts is a cut).
2. From your homework assignments: all mandatory exercises (oral or written) that were listed in the first five assignments.
3. From the lecture: How all rational numbers may be constructed from all integers, similarities and differences between statements shown in class and Theorems 37, 38, 39, 40, 68, 69, 70, and 71.

You should also remember all other Theorems stated in the lecture, or contained in the first five chapter of Landau's proof even if you are not expected to remember their proofs.

3 What to expect

The exam will be *closed book*, the only handout you may use is the list of the Zermelo-Fraenkel Axioms. You will have 80 minutes to answer about 10 questions. Some questions may ask you to state and prove a theorem from the list above, others may be like the exercises from your homework assignments. There may be questions where you have to decide about an example whether it has certain properties. (E.g. "Is this ordered set well-ordered?" "Is this subset of \mathbb{Q}^+ a cut?" etc.)