Study Guide for the Final

1 Definitions and axioms to remember

1. From the yellow book: Definition 3.3 (absolute value), 4.1 (minimum and maximum), 4.2 (upper and lower bounds), and 4.3 (supremum and infimum). Axiom 4.4 (completeness). Definition 7.1 (convergence), 9.8 (∞ as a limit), 10.1 (monotone sequences), 10.6 ($\liminf$ and $\limsup$), 10.8 (Cauchy sequences), 11.1 (subsequences), 11.6 (subsequential limits), 14.1 (summation notation), 14.2 (infinite series), 14.3 (Cauchy criterion for series).
2. From your notes: accumulation point of a set.

2 Theorems you should remember

1. With proof: Theorem 3.5/(iii) (triangle inequality, remember also the generalization in Exercise 3.6), Corollary 4.5 (existence of inf), Properties 4.6 and 4.7 (Archimedean property and denseness of $\mathbb{Q}$, using the Completeness Axiom), Theorem 9.1 (convergent sequences are bounded), Theorems 9.2, 9.3, and 9.6 (what happens to the limit when you multiply by a constant, add or divide two sequences), Examples 9.7/a,b, Theorem 10.2 (monotone bounded sequences converge), 10.4 (monotone unbounded goes to $\pm\infty$), Corollary 10.5, Lemma 10.9 (convergent sequences are Cauchy sequences), Lemma 10.10 (Cauchy sequences are bounded), Theorem 11.3 (every sequence has a monotonic subsequence), Theorem 14.4 (Cauchy criterion for series, only equivalence with sequential Cauchy criterion), Example 14/1 (geometric series), Corollary 14.5 (terms in a convergent series go to zero), 14.8 (Ratio Test, proof only of weaker form in Exercise 9.12/a).
2. Without proof: Theorems 9.4, 9.5, 9.9, and 9.10 (limit of product and inverse of a sequence), Examples 9.7/c,d, Theorem 10.7 (limit exists iff $\limsup$ is same as $\liminf$), Theorem 10.11 (Cauchy sequences converge), Theorem 11.2 (subsequences have the same limit), Corollary 11.4 (equivalent definition of $\limsup$ and $\liminf$), Theorem 11.5 (Bolzano-Weierstrass), Theorem 11.7 (set of subsequential limits), Theorem 12.1 ($\limsup$ of a product), Example 14/2 ($\sum_{n=1}^{\infty}(1/n^p)$), 14.6 Comparison Test, 14.7 Corollary (absolute convergent implies convergent), 14.9 (Root test).

3 What to expect

The exam will be *closed book*, you may use the same handouts as for the midterm. The above guide is meant to help with the mandatory part. For the optional part prepare as if it was another midterm. The mandatory part will be as long as the midterm, the optional part will have only about 5 questions.

Besides remembering the definitions, theorems, and proofs above, you should be prepared to calculate the limit of a simple sequence or series using the definition of convergence only, or the limit of a more complicated sequence or series, using all the results we learned. I could also ask you to determine the supremum, the infimum and the accumulation points of a set, or the set of subsequential limits of a sequence.