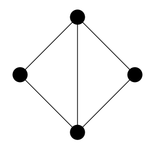
Name: **Student ID:**

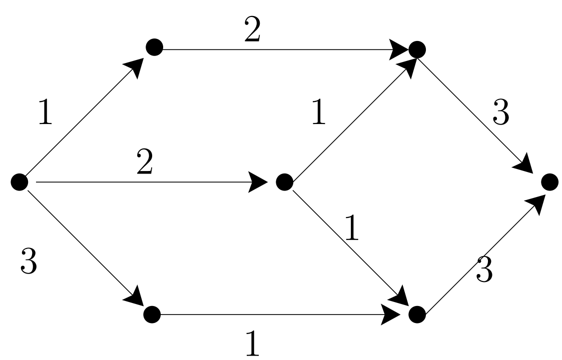
Sample Test 2

The actual test will have only 5 *questions.*

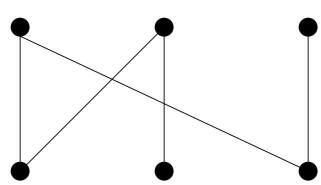
1. Give an upper bound for the number of edges of a plane graph in terms of its girth and the number of its vertices, and prove your claim.
2. Prove that K^5 is not planar.
3. Prove that $K_{3,3}$ is not planar.
4. State Kuratowski's theorem.
5. Find the plane dual of the graph shown below.



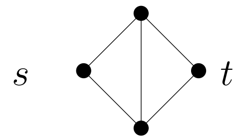
6. Which theorem is used to prove that a graph is planar if and only if it has an abstract dual?
7. Let G be a plane graph, G^*, its dual, and e an edge of G that is neither a bridge, nor a loop. How can you get the dual of $G \setminus e$ and the dual of G/e from G^*?
8. Find a minimum cut and a maximum flow for the graph shown below. **Show all your work!**



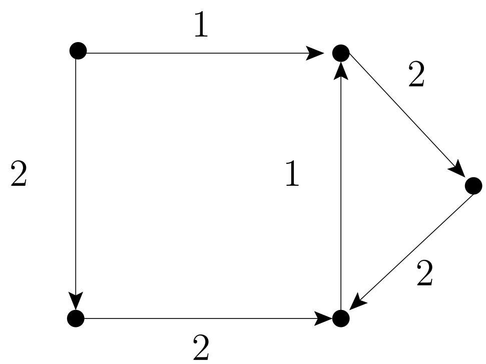
9. Find a maximum size matching and a minimum size cover for the bipartite graph below, by converting the problem to a maximum flow-minimum cut problem. **You will get full credit only if you find the answer using network flows.**



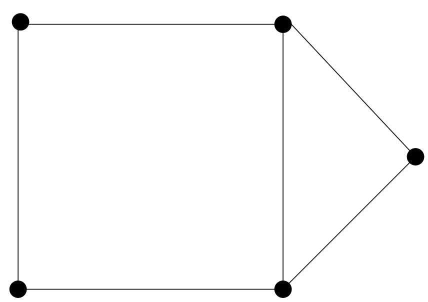
10. State Menger's theorem on the maximum number of internally disjoint paths between two vertices. Illustrate how this statement may be reduced to the Ford-Fulkerson theorem, using the graph below.



11. Convert the $\mathbb{Z}_3$-flow shown below to a 3-flow.



12. Find the polynomial expressing the number of H-flows in terms of $|H|$ for the graph below.



Good Luck. Gábor Hetyei