GPU-Accelerated Network Centrality

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GTC 2015
Outline

1 Introduction
2 Decomposition for GPU
3 An SpMM-based approach
4 Conclusion
Centralities - Concept

Answer questions such as
- Who controls the flow in a network?
- Who is more important?
- Who has more influence?
- Whose contribution is significant for connections?

Different kinds of graph
- road networks
- social networks
- power grids
- mechanical mesh

Applications
- Covert network (e.g., terrorist identification)
- Contingency analysis (e.g., weakness/robustness of networks)
- Viral marketing (e.g., who will spread the word best)
- Traffic analysis
- Store locations
Centrality Formally

Closeness Centrality
Let $G = (V, E)$ be an unweighted graph with the vertex set $V$ and edge set $E$.

$$cc[v] = \sum_{u \in V} \frac{1}{d(v, u)}$$

where $d(u, v)$ is the shortest path length between $u$ and $v$.

Betweenness Centrality
Let $G = (V, E)$ be an unweighted graph. Let $\sigma_{st}$ be the number of shortest paths connecting $s$ and $t$. Let $\sigma_{st}(v)$ be the number of such $s$-$t$ paths passing through $v$.

$$bc[v] = \sum_{s \neq v \neq t \in V} \delta_{st}(v)$$

where

$$\delta_{st}(v) = \frac{\sigma_{st}(v)}{\sigma_{st}}.$$

Algorithm
In each case, the best algorithm computes the shortest path graph rooted in each vertex of the graph and extract the relevant information. The complexity is $O(E)$ per source, $O(VE)$ in total, which makes its computationally expensive.
Computing Breadth First Traversal (Centrality)

Top-down (scatter writes)
For each element of the frontier, touch the neighbors.
Complexity: $O(E)$
 Writes are scattered in memory

Bottom-up (gather reads)
For each vertex, are the neighbors in the frontier?
Complexity $O(ED)$, where $D$ is the diameter of the graph.
 Writes are performed once linearly.

Direction Optimizing.
Level synchronous bfs.
Outline

1. Introduction
2. Decomposition for GPU
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Traditionally ...

Vertex Centric

<table>
<thead>
<tr>
<th>1 thread: 1 vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>adj</td>
</tr>
<tr>
<td>ptrs</td>
</tr>
</tbody>
</table>

- No graph coalescing
- Vector read is not coalesced
- No atomics
- High divergence (high degree)
Traditionally...

**Edge Centric**

1 thread: 1 edge

- Graph read is coalesced
- Vector read is not coalesced
- Many atomics
- Little divergence (likely to have adjacent thread doing the same vertex)
Virtual vertex decomposition

Virtual Vertex

1 thread: 1 virtual vertex

- No graph coalescing
- Vector read is not coalesced
- Some atomics
- Bounded divergence

High degree vertices are split in multiple "virtual vertices"
### Strided virtual vertex decomposition

- **Virtual Vertex**

  **1 thread: 1 virtual vertex**

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<tbody>
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<td>2</td>
<td>3</td>
<td>4</td>
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<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

- **Some graph coalescing**
- **Vector read is not coalesced**
- **Some atomics**
- **Bounded divergence**

![Memory: \(2n' + n + m + 1\)](image)
Experimental Setting

Instances

| Graph         | |V| | |E| | Avg|Γ(v)| | Max|Γ(v)| | Diam. |
|---------------|---|---|---|---|---|---|---|---|---|---|---|
| Amazon        | 403K | 4,886K | 12.1 | 2,752 | 19 |
| Gowalla       | 196K | 1,900K | 9.6 | 14,730 | 12 |
| Google        | 855K | 8,582K | 10.0 | 6,332 | 18 |
| NotreDame     | 325K | 2,180K | 6.6 | 10,721 | 27 |
| WikiTalk      | 2,388K | 9,313K | 3.8 | 100,029 | 10 |
| Orkut         | 3,072K | 234,370K | 76.2 | 33,313 | 9 |
| LiveJournal   | 4,843K | 85,691K | 17.6 | 20,333 | 15 |

Machines

- 2 Intel Sandybridge EP
- NVIDIA K20

Metric

Traversed Edge Per Second: \( \frac{VE}{time} \).
First results

Speedup wrt CPU 1 thread

- GPU vertex
- GPU edge
- GPU virtual
- GPU stride

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GPU Centrality

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No vector coalescing

All the representations give vector coalescing only “if you are lucky”
Simultaneous sources traversal

The problem with previous methods is that BFS leaves the coalescing of the access to the vector up to the structure of the graph.

Multiple sources

- All threads of a warp should make similar access pattern.
- Since there are multiple traversals to perform in a Centrality computation, process $B$ traversals at once.
- If a vertex is in the same level of the BFS in multiple traversal, they will be processed at the same time.
- Social networks have most vertices in a few levels.
An SpMV-based approach of BFS for Closeness Centrality

A simpler definition of level synchronous BFS

Vertex $v$ is at level $\ell$ if and only if one of the neighbors of $v$ is at level $\ell - 1$ and $v$ is not at any level $\ell' < \ell$.

Let $x^\ell_i = \text{true}$ if vertex $i$ is a part of the frontier at level $\ell$.

$y^{\ell+1}$ is the neighbors of level $\ell$. $y^{\ell+1}_k = \text{OR}_{j \in \Gamma(k)} x^\ell_j$. ( (OR, AND)-SpMV )

Compute the next level frontier $x^{\ell+1}_i = y^{\ell+1}_i \& \neg (\text{OR}_{\ell' \leq \ell} x^{\ell'}_i)$.

Contribution of the source to $cc[i]$ is $\frac{x^\ell_i}{\ell}$.

It allows to compute Closeness Centrality by encoding the state of 32 traversal with an int.
Impact on working warps

Number of active warps necessary for 32 traversals. Small increase in the number of warps.
With $B = 4$, 32 traversals of one vertex are distributed in about 40% of 32 warps. Good coalescing.
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On other architecture? Betweenness Centrality

\(O(DE)\) algorithms (GPU-) is unsuitable for NotreDame because of its high diameter.

CPU: 2 Intel Sandybridge EP (2x8 cores)
On other architecture? Closeness Centrality

CPU: 2 Intel Sandybridge EP (2x8 cores)
PHI: Intel Xeon Phi 5120

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Conclusion

Centrality

Betweenness and Closeness Centrality are computed using multiple Breadth First Search traversal.

Graph representation for GPU

- Vertex Centric
- Edge Centric
- Virtual Vertex
- Coalesced Virtual Vertex

Determine parallelism but also memory access patterns and thread divergence.

Multiple traversals

- Centrality requires graph traversal from many different sources.
- Threads of a warp can be set to process different traversal for the same decomposition.
- Provided a vertex is used in the same level in multiple traversals, all the memory accesses can be coalesced.

Improves performance by a factor of 70x. Adapts to CPU architecture for similar effects.
Thank you

Other centrality works (with Sarıyüce, Kaya and Çatalyürek)

- Compression using graph properties (SDM 2013)
- GPU optimization (GPGPU 2013)
- Incremental algorithm (BigData 2013)
- Distributed memory incremental framework (Cluster 2013, ParCo 2015)
- Regularized memory accesses for CPU, GPU, Xeon Phi (MTAAP 2014, JPDC 2015)

More information

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