Parallel Dataflow Graph Coloring

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Scheduling in Aussois\textsuperscript{W} Dagstuhl\textsuperscript{W}\textsuperscript{W}\textsuperscript{W}\textsuperscript{W}
Algorithms and Scheduling Techniques for Exascale Systems
Outline

1. Parallel Graph Coloring
2. Dataflow Graph Coloring
3. What’s the link with scheduling?
4. Conclusion
The Graph Coloring Problem

**Definition**
Coloring a graph consists in assigning a color (an integer) to each vertex so that no two adjacent vertices have the same color.

**Complexity**
The problem of finding the coloring with minimum number of colors is NP-Hard.
No approximation within $|V|^{1-\epsilon}$. Greedy algorithm returns a solution with less than $1 + \Delta$ colors.
Graph Coloring Algorithm

First Fit algorithm

Pick a vertex and assign it the first available color. Then pick another one.
There exists a vertex ordering which leads to an optimal coloring.

Algorithm 1: Sequential greedy coloring.

Data: \( G = (V, E) \)

for each \( v \in V \) do
  for each \( w \in \text{adj}(v) \) do
    forbiddenColors[\text{color}[w]] ← \( v \)
  color[\( v \)] ← \( \min \{ i > 0 : \text{forbiddenColors}[i] \neq v \} \)

Many derivative algorithms:
- With Largest First
- With Smallest Last
- Dynamic orderings
- Least Used instead of First Fit.
- Iterated algorithm to do local descent.

Today, let’s talk about the natural one.
Parallel Speculative Graph Coloring (Shared Memory)

Algorithm 2: TentativeColoring

Data: $G = (V, E)$, $\text{Visit} \subset V$, $\text{color}[1 : |V|]$

1. $\text{maxcolor} \leftarrow 1$
2. $\text{localMC} \leftarrow 1$
3. for each $v \in \text{Visit}$ in parallel do
   3.1. for each $w \in \text{adj}(v)$ do
      3.1.1. $\text{localFC}[\text{color}[w]] \leftarrow v$
      3.1.2. $\text{color}[v] \leftarrow \min\{i > 0 : \text{localFC}[i] \neq v\}$
   3.2. if $\text{color}[v] > \text{localMC}$ then
      3.2.1. $\text{localMC} \leftarrow \text{color}[v]$
5. $\text{maxcolor} \leftarrow \text{Reduce}(\text{max}) \text{localMC}$
6. return $\text{maxcolor}$

Algorithm 3: DetectConflict

Data: $G = (V, E)$, $\text{Visit} \subset V$, $\text{color}[1 : |V|]$

1. $\text{Conflict} \leftarrow \emptyset$
2. for each $v \in \text{Visit}$ in parallel do
   2.1. for each $w \in \text{adj}(v)$ do
      2.1.1. if $\text{color}[v] = \text{color}[w]$ then
      2.1.2. if $v < w$ then
      2.1.3. atomic $\text{Conflict} \leftarrow \text{Conflict} \cup \{v\}$
3. return $\text{Conflict}$

At least two passes. More if unlucky (in practice $2 + \epsilon$)
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Parallel Dataflow Algorithm

**Principle**
The principle of Dataflow algorithm is that the generation of a result triggers the computation of the next tasks.

**Dataflow coloring**
The idea is to pick an absolute order of the vertices and each vertex only consider the color of the vertices with ID lesser than theirs.

- 0 and 1 can be executed concurrently
- 2 and 3 can be executed concurrently
- 4 and 5 can be executed concurrently

Not speculative, so only one pass.
Two approaches

Pick the vertices in some order. What happens when you pick a vertex with neighbors with high priority which haven’t been allocated a color.

**Recursive Dataflow**
You recursively process the neighbor.
- No waiting time
- Some form of “workstealing” algorithm
- Complex synchronisation
- Higher memory allocation (or potentially redundant work)

**(Direct) Dataflow**
You wait.
- No redundant work
- Simpler worksharing constraint.
- But maybe you waste time waiting.
Which is best?

with lots of (yet) unexplained optimizations.

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In practice, parallel speedup is 1

The graph is executed one vertex after another. So there are actually dependencies.

Graham List Scheduling

When scheduling a dag, a greedy algorithm gets:

\[ C_{max} \leq \frac{W}{p} + (1 - \frac{1}{p}) CP \]
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It gets worse...

If you use a static OpenMP schedule, you add \textit{de facto} dependencies in your graph. And the critical path increases significantly.

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If you use a static OpenMP schedule, you add *de facto* dependencies in your graph. And the critical path increases significantly.

That’s easy! Let’s use dynamic instead!
Even if you use a dynamic OpenMP schedule, similar effect still happen. With two threads 4 and 5 need to be executed before 6 can start. So 1, 2 and 3 are implicit predecessor of 6. Because that is what the scheduler will do.

An easy solution
Compute a level by level order. Well... That requires a graph traversal. The whole point was to traverse the graph only once.
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Nobody should use “dynamic,1”

Since vertices are grouped together, by openmp’s granularity, you have implicit edges between the vertices of each group. There is an implicit edge between 1 and 2, and between 5 and 6.

In this type of kernel, you should at least use groups of 32 vertices.
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First Results

(a) auto

(b) ldoor

Ouch!
Reordering of vertex IDs

Just a chain

1 2 3 4 5 6 7 8

The critical path can be quite long in a natural ordering.

At best

1 2 3 4
5 6 7 8

Need to traverse the graphs...
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At best

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5 6 7 8

Need to traverse the graphs...

At random

6 2 1 4 3 7 5
8

Not best but probably not the worst you can get.
For cache purposes, you need to keep some locality, so you shuffle blocks of vertices.
Any guarantee on that?
Results

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Parallel Dataflow Coloring

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Graph showing the speedup of different shuffle block sizes with varying numbers of threads.
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Faster than speculative coloring?

Owens (8 cores)

Oakley (16 cores)

Works on small machines.
Impact seem to decrease with core count.

- There is an other limiting factor

Mirasol (40 cores)
Wrap up

Conclusions

- designed and tested dataflow coloring algorithms on multicore architectures.
- analyzed their performance.

Classical scheduling helps understanding performance issues in not-so-related problems. Similar analysis performed on BFS.

Future works

- how to pick a better order?
- can we get rid of the computation and lookup of permutation?
Thank you

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