Game Theoretic Approach for Joint Power Control and Routing in Wireless Sensor Networks

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Abstract—We propose a game theoretic approach for joint power and parent adaptation for wireless sensor networks to improve the network lifetime. A data collecting sensor network is assumed that employs asynchronous duty-cycling for energy conservation. In such networks, overhearing dominates the energy consumption, which can be controlled by adapting the transmit power levels as well as the route selections. The goal of this work is to determine whether there exists an assignment of power levels and parent selections for all sensors so that each sensor’s route to sink has an acceptable quality (ETX) and also maximizes the lifetime of the network by controlling the overhearing in the nodes. Results from computer simulations are presented to demonstrate the performance of this approach.

I. INTRODUCTION

Energy management is a key requirement for achieving long term survivability of wireless sensor networks (WSN). This is due to the difficulties in battery replacement in the sensor nodes, especially when deployed in remote locations. Consequently, the nodes must optimize the usage of their onboard energy resources (battery) to maximize their lifetime. All functions in WSNs, including communication protocols for multihop wireless networking, sensing/monitoring tasks, multi-sensor collaboration, localization, time-synchronization, etc. must be performed with minimum wastage of energy resources. A viable approach for addressing the energy problem is to harvest energy from the environment, such as solar, vibrations, thermal, etc., which potentially provides the possibility of unlimited lifetime. However, WSNs that are powered by environmental energy also face many design challenges for energy management. Firstly, the harvested energy must be stored in onboard rechargeable batteries for continuous operation, which have a finite cycle life, i.e. number of charge-discharge events possible. Hence, charging and discharging must be conserved. Secondly, the energy harvested at the nodes experience large variations over space and time, due to natural and uncontrollable differences of ambient environmental energy availability. For instance, solar energy highly depends on the time of day, solar irradiance patterns, weather, as well as location-specific shading factors. Consequently, it is critical for the sensor nodes powered by environmental energy to adapt the energy consumption depending on its current energy resources as determined by the state of charge of its rechargeable batteries.

In this work, we consider application of adapting power control and routing to control the energy consumption in the sensor nodes based on their individual energy resources. We assume WSNs with data collection traffic that is typical of environmental monitoring applications where all sensor nodes transmit periodic sensor readings to a centralized sink node. We further assume that the network applies asynchronous duty cycling of sleep and wake states of the radio, also known as low-power listen (LPL), which is a popular technique for conserving energy without requiring network wide time-synchronization, which has additional overhead. In such networks, overhearing is a dominating factor in the energy consumption at the sensor nodes. Several approaches have been proposed to reduce this overhearing effect by appropriate modifications of the LPL format, such as interruption of reception of unnecessary packets based on information transmitted in the preamble [1], adaptive duty-cycling [2], [3] and others. However, these techniques do not completely eliminate the energy wastage from overhearing in asynchronous networks, which can be a significant loss in large scale and high density WSNs.

In our earlier work [4], we demonstrated that joint power control and routing can effectively control the energy consumption by controlling the overhearing effect. However, this requires cooperative control under a given set of energy constraints, which is difficult to solve in large scale networks. Here we address the problem using a game theoretic framework. The main contributions of this work are as follows. First, we prove that the proposed distributed power control and routing problem can be posed as a finite exact potential game when the potential function is defined using a combination of the network lifetime and the end-to-end route quality. Secondly, we evaluate the lifetime improvement that can be achieved using our game theoretic formulation, which requires global information and can be only be achieved using centralized computations. However, results from such centralized approaches are important to estimate the "best case" performance that can be achieved with joint power control and routing that is numerically intensive. Finally, we evaluate a distributed implementation of the power control and routing algorithm, and present performance evaluations obtained from simulations.

1We consider the network lifetime to be the smallest lifetime of the nodes in the network under their current conditions of battery charge and current consumptions.
II. PRELIMINARIES

We assume that each node $S_i$ in the network is able to estimate its remaining battery capacity $B_i$ as well as its average current consumption $I_i$ under its current set of activities. For data collecting networks with constant data rate and duty cycle, the average current can be represented as [5]:

$$I_i = \frac{I_B T_B}{2} + M I_{Dx} T_{Dx} + N I_B T_B + O I_{Dx} T_{Dx} + F I_{Dx} T_{Dx} + \frac{I_P T_P}{2} + N_P I_P T_P$$

where $I_B$ and $T_B$ represent the current drawn and the duration, respectively, of the event $x$; and $T_{Dx}$ represents the beacon interval. Transmission/reception of beacons is denoted by $B_i/B_r$, data transmit/receive is denoted by $D_i/D_r$, and processing and sensing are denoted as $P$ and $S$, respectively. $O$ and $F$ are the overhearing and forwarding rates, respectively, and $N$ is the number of neighbors. $M$ is the rate at which a node transmits its own packets. If there are no retransmissions, then $M = \frac{1}{T_B}$, where $T_{Dx}$ is the data interval. $N_P$ represents the number of times that a node wakes per second to check whether the channel is busy, and is set to 8 in our application. We assume that each node is able to estimate all the dynamic parameters that are used in equation (1), by periodic assessments of its overhead and forwarded traffic.

With this, the remaining lifetime of $S_i$ can be written as:

$$L_i = \frac{B_i}{T_i}$$

Our objective is to determine mechanisms to control the $I_i$ terms for a given distribution of battery estimates $B_i$, so that the network lifetime, i.e. $\min \{L_i\}$ is maximized. This is achieved by methods that reduce the current consumption of the node with the minimum $L_i$, iteratively. We term the node with the lowest $L_i$ as the critical node (note that the critical node changes from time to time). The principle for joint power control and routing for controlling the average current in the critical node is illustrated in Figure 1. Here, one of the neighbors of the critical node (marked in red) reduces the transmit power to reduce the overhearing to the critical node. Since overhearing is a dominating factor in the current equation (equation 1), this helps in reducing the average current consumption in the critical node. Note that this might require the neighboring node to find a new parent in order to be able to transmit to the sink using the lower transmit power. In addition, we also perform overhearing control by controlling the amount of forwarding traffic carried by neighbors of critical nodes. This can be achieved by selectively diverting the routes of children of neighbors of critical nodes to regions that exclude the neighborhood of the critical node.

The main challenge for implementing this approach is to determine an optimal policy for selecting power levels and routes in the network, given a random distribution of battery levels of the nodes. This task is computationally complex even with global information. In the next section, we pose this problem as a multi-player game to obtain the best case results using global information.

III. GAME THEORETICAL FORMULATION OF JOINT POWER AND PARENT ADAPTATION PROBLEM

Consider a wireless sensor network of $n$ sensors. Since the lifetime of the WSN is determined by the lowest lifetime of the nodes, we consider approaches for improving the lifetime of the critical node only. Our approach for doing this is by reducing the overhearing at the critical node caused by its neighbors, which is done by reducing the power level of neighbors (i.e., shortening the transmission range by selecting a new parent). Since this potentially changes the network topology, power control is associated with route selection as well, i.e. it leads a joint power control and route selection problem for improving the lifetime of the critical node.

We use game theory to address this problem, which is justified due to the following reasons:

- First, the sensors make decisions spontaneously and independently to maximize their own payoffs. They compete to achieve that. Their objectives may be conflicting as every node selfishly tries to maximize its own payoff.

- Secondly, the decision of one sensor node may influence or have an effect on other nodes. Nodes can cooperate to maximize the global objective.

A family of potential games were introduced by Monderer and Shapley [6]. These games received increasing attention recently in the field of wireless networks [7], [8], [9]. We formulate the proposed power control and routing in energy constrained WSN as a strategic game, which is proven to be an exact potential game. Consequently, it has a pure strategy Nash equilibrium (NE), which is our desired result for a centralized solution for the problem. Formally, the game is denoted by $\mathcal{G} = (S, \{A_i\}_{S \in S}, \{u_i\}_{S \in S})$. The normal form (or strategic form) representation of the proposed game consists of following elements:

- A set of players $S = \{S_1, S_2, ..., S_n\}$, which is a group of nodes in a given wireless sensor network.

- A set of actions $A_i = \{a_1, a_2, ..., a_m\}$ is the set of available actions for player $S_i \in S$. $A = A_1 \times ... \times A_n$ is the space of all possible joint actions. The strategy profile is the pair: parent and power level. Hence, $a_i \in A_i$ represents the parent and power level selection of node $S_i$, and $a_{i-1} \in A_1 \times ... \times A_{i-1} \times A_{i+1} \times ... \times A_n$ represents a parent and power selection profile of all
the nodes excluding $S_i$, where $\times$ denotes the Cartesian product.

- The payoffs $\{u_1, u_2, ..., u_n\}$ resulted from the strategy profile. For each player $S_i \in S$ a payoff function $u_i : A \rightarrow R$, where $A = \times_{S \in S} A_i$.

A node can choose different actions to maximize the outcome. The optimal outcome of a game is one where no player has an incentive to deviate from its chosen strategy after considering other’s choice. An NE exists when there is no unilateral profitable deviation from any of the players involved. To reach Nash Equilibrium, it is truly necessary to maximize the utility function. If the incentives of all players in the game can be expressed in one global function, then the game is called a potential game. In potential games, the incentives of all players are mapped into one function called the potential function. The pure NE can be found by locating the local optima of the potential function [10].

**Definition 1.** (Potential game). A game $\zeta$ is a potential game (ordinal and exact) if there exist a function $\phi : A \rightarrow R$ such that $\phi(a_i, a_{-i})$ gives the information about $U_i(a_i, a_{-i})$ for each $S_i \in S$.

The potential function is a real valued function on the strategy space that matches a deviation to a change of the potential value. Depending on the matching, a game can be an ordinal potential game or an exact potential game. In exact potential game, the difference in payoffs of a node when changing unilaterally its strategy has the same value as the difference in values of the potential function. Whereas, in ordinal potential game, only signs of differences have to be the same.

**Ordinal potential game:** A function $\phi : A \rightarrow R$ is an ordinal potential for game $\zeta$ if for every $S_i \in S$ and for every $a_{-i} \in A_{-i}$:

$$u_i(x, a_{-i}) - u_i(z, a_{-i}) > 0 \text{ iff } \phi(x, a_{-i}) - \phi(z, a_{-i}) > 0$$

$\forall x, z \in A_i$. In other words, if player $S_i$ acquires a better (worse) payoff by unilaterally deviating from one strategy to another, the potential function will also increases (decreases) with this deviation.

**Exact potential game:** A function $\phi : A \rightarrow R$ is an exact potential for game $\zeta$ if for every $S_i \in S$ and for every $a_{-i} \in A_{-i}$:

$$u_i(x, a_{-i}) - u_i(z, a_{-i}) = \phi(x, a_{-i}) - \phi(z, a_{-i})$$

$\forall x, z \in A_i$.

In other words, a normal form game is called an exact potential game if there exists a potential function which reflects the exact change in the utility received by every unilaterally deviating player.

A new class of potential games, namely best response potential games, were studied in [11]. The distinctive feature is that it allows improvement paths, by imposing restrictions only on paths in which players that can improve actually deviate to a best response.

**Definition 2.** (Best response potential game). A game $\zeta$ is a best response potential game if there exist a function $\phi : A \rightarrow R$ such that for every $S_i \in S$ and for every $a_{-i} \in A_{-i}$:

$$\arg \max_{a_i \in A_i} u_i(a_i, a_{-i}) = \arg \max_{a_i \in A_i} \phi(a_i, a_{-i})$$

We define the utility function of node $S_i$ corresponding to action $a_i$ in the proposed game as:

$$u_i(a_i) = W_i * f(\mathcal{L}_c) + W_{ETX} * f(pathETX_i)$$

(5)

where, $u_i(a_i)$ is the utility of sensor $S_i \in S$ for action $a_i$, $\mathcal{L}_c$ is the lifetime of critical node, $pathETX_i$ is the path ETX as used in the collection tree protocol (CTP) [2] of sensor $S_i$ for selecting a new parent, and $W_i$ and $W_{ETX}$ are weight factors for the network lifetime and path ETX, respectively.

Similarly, the potential function of the proposed game is defined as

$$\phi(a) = W_i * f(\mathcal{L}_c) + W_{ETX} * \sum_{S_i \in S} f(pathETX_i)$$

(6)

Then the system-centric objective is to find the optimal parent selection profile $a_{opt}$ such that the system throughput is maximized. Formally,

$$a_{opt} = \arg \max \phi(a)$$

(7)

**Theorem 1.** The joint power route adaptation game is a finite exact potential game with potential function $\phi$ and it has at least one pure strategy Nash equilibrium.

**Proof:** We need to prove that $\phi$ is an exact potential function in parent assignment game which satisfies equation 4. Assuming that a player $S_i \in S$ changes its parent to $k' \rightarrow k$, it makes the action changes to $a_i$ from $a_i$, while others are using the strategy profile $a_{-i}$. The difference of utility function is:

$$u_i(a_i', a_{-i}) - u_i(a_i, a_{-i}) = W_i * (f(\mathcal{L}_c') - f(\mathcal{L}_c)) + W_{ETX} * (f(pathETX_i') - f(pathETX_i))$$

(8)

Now consider the potential function (6), which can be expressed as:

$$\phi(a_i', a_{-i}) = W_i * f(\mathcal{L}_c') + W_{ETX} * f(pathETX_i') + \sum_{S_i \in S, S_i \neq S_i} f(pathETX_i')$$

The difference in potential function is:

$$\phi(a_i, a_{-i}) - \phi(a_i', a_{-i}) = W_i * (f(\mathcal{L}_c') - f(\mathcal{L}_c)) + W_{ETX} * (f(pathETX_i') - f(pathETX_i)) + \sum_{S_i \in S, S_i \neq S_i} (f(pathETX_i') - f(pathETX_i))$$

Let,

$$d_{ETX} = W_{ETX} * \sum_{S_i \in S, S_i \neq S_i} (f(pathETX_i') - f(pathETX_i))$$

Now if we limit only one node can change its parent at a time, say $S_i$, then the difference in potential function becomes:

$$\phi(a_i', a_{-i}) - \phi(a_i, a_{-i}) = W_i * (f(\mathcal{L}_c') - f(\mathcal{L}_c)) + W_{ETX} * (f(pathETX_i') - f(pathETX_i)) + d_{ETX}$$

(9)
Now, substituting equation 8 into equation 9, we get:
\[
\phi(a'_i, a_{i-1}) - \phi(a_i, a_{i-1}) = u_i(a'_i, a_{i-1}) - u_i(a_i, a_{i-1}) + d_{ETX}
\] (10)

If we restrict only one node can change its parent at a time then \(d_{ETX} = 0\) in equation 10. Thus, \(\phi\) becomes an exact potential function.

Clearly, any exact potential game is an ordinal potential game (if \(d_{ETX} \neq 0\)) but not the other way around. Here each node selects its best response and it is defined as:
\[
B_i(a_{i-1}) = \arg \max_{\forall a_i \in A_i} u_i(a_i, a_{i-1})
\] (11)

Best response dynamics is an update rule where at each time instant a player chooses its best response to other players’ current strategy profile.

In any finite potential game, best response dynamics always converge to a Nash equilibrium [10], [11]. The global maximum of a ordinal potential function is a pure strategy Nash equilibrium. To understand this, let \(a^*\) corresponds to the global maximum. Then, for any \(S_i \in S\), we have, by definition,
\[
\phi(a^*, a^*_{i-1}) - \phi(a_i, a^*_{i-1}) \geq 0, \forall a_i \in A_i
\]

But as \(\phi\) is a potential function, for all \(S_i \in S\),
\[
u_i(a^*, a^*_{i-1}) - u_i(a_i, a^*_{i-1}) \geq 0 \quad \forall f f
\]
\[
\phi(a^*, a^*_{i-1}) - \phi(a_i, a^*_{i-1}) \geq 0, \forall a_i \in A_i
\]

Therefore, in best response dynamics:
\[
u_i(a^*, a^*_{i-1}) - u_i(a_i, a^*_{i-1}) \geq 0, \forall S_i \in S \quad \text{and} \quad \forall a_i \in A_i
\] (12)

Hence \(a^*\) is a pure strategy Nash equilibrium. However, there may also be other pure strategy Nash equilibria corresponding to local maxima. Note that implementing this scheme would require all nodes to have global knowledge of node parameters, which will lead to heavy communication cost and is unrealistic in practice. Our objective of the game theoretic formulation is two fold. Firstly, it provides the best case results for a set of probable parents (utility \(\vec{u}_i\)), which is later used to select best response. A profile of power adaptation and new parent selection (selectParent\(S_i, u_i, \vec{a}_i\)) strategies results in a profile of expected utility or payoffs. Nodes which have non zero utility vector participate in the game in that round. Let at any given round there are \(k\) such nodes. We consider that a subset \(l\) out of these \(k\) nodes take action to improve the critical node lifetime. This is achieved by using a probability \(p\) for each node to take action, where \(p = (L_a - L_c)/(\alpha \times k \times L_a)\); \(L_a\) is the average lifetime of the network; \(L_c\) is the lifetime of the critical node in the network; and \(\alpha\) is a adjustment factor.

In the next round each node’s strategy is determined by its best response (bestResponse\(u_i\)) to this selected subset (i.e., \(l\) nodes) of the population. This approach maximizes each nodes payoff with respect to its strategy.
value is also calculated by the sink to see the improvement in the network. After taking action, each node calculates its new payoff and explores if there is any chance of improving the utility. To ensure system stability, it is important to know whether the game converges to an equilibrium. In the proposed scheme, a sensor node selects a parent (and adjusts the transmission power level) so as to maximize its own utility. Thus every sensor is playing its best strategy. The game reaches an equilibrium state if there is no such node which can improve its utility. If the game reaches NE, then change of any nodes profile disturbs the equilibrium state (NE). Thus there is no further advantage of changing the strategy. However, the selfish behavior by sensors may lead to an inefficient result. This could be improved upon given dictatorial control over every nodes actions. On the other hand, imposing such control can be costly or infeasible (due to the oscillation in the states) with large networks. Therefore, it is significant to find the conditions under which decentralized optimization by sensors is guaranteed to produce a near-optimal outcome.

IV. DISTRIBUTED POWER CONTROL AND ROUTING SCHEME FOR IMPROVING NETWORK LIFETIME (DPCR)

We now present the proposed DPCR algorithm that requires a minimum amount of global information from the network, and hence can be implemented without incurring heavy communication cost.

Initially routes are setup using CTP. All nodes transmit their health metric (lifetime) to sink. The sink then selects \( k \) nodes with the lowest lifetimes (critical node set) \( CN = \{CN_1, CN_2, \ldots, CN_k\} \), and broadcasts that to the network. Let \( N^c \) be the set of nodes that causes overhearing to any critical node. Then all \( S_j \in N^c \) and its child nodes are active nodes in this game. Every active node in the network calculates its utility value, which is a function of the overhearing caused to any member of \( CN \) en-route to the sink (OH), the path ETX and a cost associated with every link. The utility of node \( S_i \) is

\[
W_i = W_{oh}(f(OH)) + W_{ETX}(f(ETX)) - Cost
\]

This is similar to the utility function used in the centralized algorithm with two important differences. First, it uses \( OH \), which is the overhearing caused by any node on the current route to a member of \( CN \). This can be easily propagated by the route update packets. Secondly, it adds a \( Cost \) parameter that is used to provide additional control for parent selection under certain circumstances that is explained below. The \( Cost \) value is assigned to all \( S_j \in N^c \) and their children nodes down the tree. The initial value of \( Cost \) is zero.

Each neighbor of the critical nodes checks with their probable parents if they can improve their utility value by avoiding any overhearing caused to the neighboring critical node using power control and route adaptation. If a neighbors cannot improve its own utility, i.e. cannot reduce the \( OH \) caused to its neighboring critical node, then it sets its \( Cost \) to value greater than zero. All nodes, including children of the neighbors of the critical nodes, perform the same tasks for parent selection. \( W_{oh} \) is different for all \( k \) critical nodes, depending on their lifetime. So, \( W'_{oh} = \frac{L_{min}}{L_i} \), where \( L_{min} \) is the minimum critical node lifetime and \( L_i \) is the lifetime of node \( i^{th} \) critical node. This will ensure that node with the lowest lifetime experiences highest overhearing as compared to any other node, given same number of packets are forwarded by their neighbor. Only a few child node participate in the game, based on a probability \( (p) \), which is defined as: \( p_i = \frac{L_{avg} - L_i}{L_{avg}} \), where \( L_{avg} \) is the average critical lifetime.

Each node selfishly tries to maximize their own payoff and its effect over the network is reflected by the potential function. This process is repeated in rounds and in every round sink broadcasts a new set of critical nodes and the utility calculation is based on these new critical nodes. NE is reached when no individual deviates from the profile while everyone else adheres to it.

V. RESULTS

The performance of the proposed DPCR scheme is obtained using computer simulations. We evaluate the network lifetime, which is given by the smallest lifetime among all nodes in the network, and the average end-to-end ETX of all routes, as obtained in every step of the process. For comparison, we also evaluate the same performance measures for the centralized schemes as well as those obtained using CTP. In our simulations, we consider a network of 40 sensor nodes that are uniformly distributed over a 50 × 50m area. All nodes transmit periodic data packets to the sink that are generated at a rate of 2 packets every 5 minutes. We evaluate the performance of the joint power control and routing schemes at an instant where the state of charge (SOC) of the batteries are randomly distributed following an uniform distribution in \((3750,5000)\) mAH. The maximum power level and transmit power levels are chosen based on parameters of a MICAz sensor node. The receiver threshold for successful packet reception in the absence of interference is assumed to be \(-90dBm\). A log-normal channel model is assumed with a reference path loss of \(-55dBm\) at 1m, a path loss exponent of 3, and a shadowing standard deviation of \(3dB\). All results are obtained by taking the average of the results obtained from 10 independent simulations with the following four sets of parameters.
same set of parameters.

Figure 2 depicts the lowest node lifetime at each step and the corresponding improvement due to the proposed scheme (centralized or DPCR) from one of the simulations runs. As expected, the centralized scheme improves the lifetime of the critical node in each step by a significant amount, which is less pronounced in the decentralized scheme (DPCR). Both schemes stabilize within 10 steps, approximately.

The average network lifetime from all simulations after each step of the algorithms are plotted in Figure 3 for all three schemes: centralized, DPCR, and CTP. It is observed that while a 30% improvement over the lifetime with CTP is achievable using the centralized approach, the improvement using the proposed DPCR is about 20%. This improvement is achieved at the cost of some reduction of network performance. This is because CTP always chooses routes that have the best end-to-end performance based on ETX values, whereas the proposed power control and routing schemes apply a combination of ETX and network lifetime criteria for power control as well as route selections. To illustrate the effect on the end to end delivery performance using the proposed schemes, we also plot the average path ETX values in Figure 4. The results indicate a slightly higher increase in the average ETX using DPCR in comparison to the centralized scheme. However, the difference is not significant.

VI. CONCLUSION

A game theoretic approach to joint power control and routing is proposed. The objective is to control the current consumption to balance the nodes remaining lifetimes, which results in improvement of the lifetime of the network. Generally, such adaptations involve a high amount of computational complexity, which is a key consideration for this work. Our approach lends an iterative solution that converges within finite time and greatly reduces the computational complexity. The problem is shown to be a finite exact potential game, which has a Nash Equilibrium. The corresponding framework is used to develop a centralized solution that is expected to give the best case results on lifetime improvement. Finally, a decentralized (albeit suboptimal) scheme DPCR is proposed, which only requires the knowledge of the list of critical nodes in the network. In DPCR, this is achieved by a single broadcast from the sink, which is required only once for running the algorithm. The lifetime improvement achieved by using DPCR as well as the average route quality in the network in comparison to the centralized approach and CTP are presented from computer simulations.

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