Low-Interference Topology Control for Wireless Ad Hoc Networks

Kousha Moaveni-Nejad, Wen-Zhan Song, Wei-Zhao Wang, Yu Wang, and Xiang-Yang Li

***Abstract*— Topology control has been well studied in wireless ad hoc networks. However, only a few topology control methods take into account the low interference as a goal of the methods. Some researchers tried to reduce the interference by lowering node energy consumption (i.e. by reducing the transmission power) or by devising low degree topology controls, but none of those protocols can guarantee low interference. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length. In this paper we give algorithms to construct a network topology for wireless ad hoc network such that the maximum (or total) link (or node) interference of the topology is either minimized or approximately minimized.**

***Index Terms*—Topology control, interference, wireless ad hoc networks.**

I. Introduction

Wireless networks have become increasingly important with the requirement for enhanced data and multimedia communications in ad hoc environments. While single hop wireless networks, or *infrastructured networks* are common, there are a growing number of applications which require multi-hop wireless infrastructure which does not necessarily depend one any fixed base-station. Wireless ad hoc network needs some special treatment as it intrinsically has its own special characteristics and some unavoidable limitations compared with wired networks. For example, wireless nodes are often powered by batteries only and they often have limited memories. A transmission by a wireless device is often received by many nodes within its vicinity, which possibly causes signal interferences at these neighboring nodes. On the other hand, we can also utilize this property to save the communications needed to send some information. Unlike most traditional static communication devices, the wireless devices often move during the communication. Therefore, it is more challenging to design a network protocol for wireless ad hoc networks. which is suitable for designing an efficient routing scheme to save energy and storage memory consumption, than the traditional wired networks. For simplification, we assume that the wireless nodes are quasi-static for a period of time.

Energy conservation is one of the critical issues in designing wireless ad hoc networks since the wireless devices here are often powered by batteries only and it is often expensive, if not impossible, to change or recharge the batteries. Many aspects of the networking will affect the energy consumption of the wireless networks, such as the physical electronic design, the medium access control (MAC) protocols, the routing protocols, and so on. Topology control, a layer between MAC and routing protocol, provides another dimension to save the energy consumption of the wireless networks. In the literature, most of the research in the topology control is about adjusting the transmission power, or designing some *sparse* network topologies that can result in more efficient routing methods. However, less attention is paid to minimize the interference caused by this structure when we perform routing on top of this structure. Notice that, if a topology has a large interference, then either many signal sent by nodes will collide (if no collision avoidance MAC is used), or the network may experience serious delay at delivering the data for some nodes, which in turn may trigger larger energy consumption.

In wireless ad hoc networks, each wireless device can selectively decide which nodes to communicate with by either adjusting its transmission power, or only maintain the communication link with some special nodes within its transmission range. Maintain a small number of communication links will also speed up the routing protocols in addition to possibly alleviate the interferences among simultaneous transmissions, to possibly save the energy consumption. The question in topology control we have to deal with is how to design a network, such that it ensures attractive network features such as bounded node degree, low-stretch factor (or called spanning ratio), linear number of links, and more importantly, low interference. In recent years, there is a substantial amount of research on topology control for wireless ad hoc networks [5], [6], [9], [10], [12], [8], [7]. However, none of these structures proposed in the literature can *theoretically* bound the ratio of the interference of the constructed structure over the interference of the respected optimum structure. A common assumption in the topology control methods is that *low node degree implies small interference*, which is not the case as shown in [3]. Notice that, in practice, almost all topology control methods will select short links and avoid longer links. However, even selecting "short" links only cannot guarantee that the interference of the resulting topology is within a constant factor of that of the optimum structure. Further, even only letting each node only connect to its nearest neighbors[1], the resulting communication graph[2] may still have an interference arbitrarily larger than the optimum, up to $O(n)$ factor. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference

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[1]Here we assume a symmetric communication. In other words, the radius r_u of a node u is set as $\max(ux, uy)$, where node u is the nearest neighbor of node x, and node y is the nearest neighbor of node u.

[2]Two nodes u and v are connected if $uv \leq \min r_u, r_v$.

is minimized while the topology is connected or is a spanner for Euclidean length.

In this paper we give algorithms to construct a network topology for wireless ad hoc network such that the maximum link (or node), or the total interference of the topology is either minimized or approximately minimized. We also study how to construct topology locally with small interference while it is power efficient for unicast routing.

The remainder of the paper is organized as follows. In Section II, we specifically discuss what is the network model used in this paper, and how we define the interference of a topology. In Section III, we proposed several methods to construct various topologies such that the maximum link interference or the total link interference of the topology is minimized. In Section IV, we proposed several methods to construct various topologies such that the maximum node interference or the total node interference of the topology is minimized. We conclude our paper in Section V by pointing out some future works.

II. Preliminaries

A. Network Model

We consider a wireless ad hoc network (or sensor network) with all nodes distributed in a two dimensional plane. It is assumed in our paper that all wireless nodes have distinctive identities and each wireless node u has a maximum transmission range R_u. The maximum transmission range of different nodes in this paper is assumed to be same and each wireless node can adjust its transmission range to any value between zero and its maximum transmission range. We only consider undirected (symmetric) communication links meaning that a message sent over a link can be acknowledged by the receiver. In other words, link uv exists if and only if the Euclidean distance between nodes u and v, denoted by $\|uv\|$, is less than R_u and R_v. It is required that the graph is connected if all nodes use their maximum transmission ranges, otherwise devising a topology that preserves the connectivity is not possible.

Energy conservation is a critical issue in wireless ad hoc networks. The energy needed to support the communication between from node u to another node v is composed three parts: (1) the energy used by node u to process the signal, (2) the energy needed to compensate the path loss of the signal from u to v, and (3) the energy needed by node v to process the signal. In the literature, the following path loss model is widely adopted: the signal strength received by a node v is p_1/r^{α}, where p_1 is the signal strength at one meter, r is the distance of node v from the source node u, and α is a path loss gradient, depending on the transmission environment. Consequently, the least signal needed to support the communication between two nodes u and v separated by distance r is $c_1+c_2r^{\alpha}$, where c_1, and c_2 are some constants depending on the electronic characteristics and the antenna characteristics of the wireless devices. Thus, we define the energy cost $c(uv)$ for each link as $c(uv) = c_1 + c_2 \cdot \|uv\|^{\alpha}$.

We also assume that the wireless devices can adjust its transmission power either to any real number from 0 to its maximum transmission power or to a given sequence of transmission powers. Furthermore, in the literature it is often assumed that each wireless device u can adjust its transmission power for every transmission depending on the intended receiver v: node u will use the minimum transmission power available to reach node v. Some researchers assume that, given a undirected network topology H, the wireless devices will only adjust its transmission power to the minimum power such that it can reach its farthest neighbor in H. In this paper, we will consider all possible power adjustments here.

B. Topology Control

Due to the limited power and memory, a wireless node prefers to only maintain the information of a subset of neighbors it can communicate, which is called *topology control*. In recent years, there is a substantial amount of research on topology control for wireless ad hoc networks [5], [6], [9], [10], [12]. These algorithms are designed for different objectives: minimizing the maximum link length (or node power) while maintaining the network connectivity [9]; bounding the node degree [12]; bounding the spanning ratio [5], [6]; constructing planar spanner locally [5]. Here a subgraph H of a graph G is a length (or power) spanner of G if, for any two nodes, the length (or power) of the shortest-path connecting them in H is no more than a constant factor of the length of the shortest-path connecting them in the original graph G. Planar structures are used by several localized routing algorithms [2]. In [11], Li *et al.* proposed the first localized algorithm to construct a bounded degree planar spanner. Recently, Li, Hou and Sha [4] proposed a novel local MST-based method for topology control and broadcasting. In [8], [7], Li *et al.* proposed several new structures that approximate the Euclidean minimum spanning tree while the structures can be constructed using local information only and with $O(n)$ total messages.

However, none of these structures proposed in the literature can *theoretically* bound the ratio of the interference of the constructed structure over the interference of the respected optimum structure. Recently, Burkhart *et al.* [3] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length.

C. What is interference?

As mentioned earlier, the ultimate goal of the topology control is to conserve the energy consumption of the wireless networks. It has been pointed out that the topology control algorithms should not only consider adjusting the transmission power of nodes, bounding the number of nodes a node has to communicate, or bounding the power spanning ratio of the structure, but also to minimize the inherent interference of the structure so multiple parallel transmissions can happen simultaneously, and the number of retransmissions is decreased. Then a nature question is "*What is the interference of a structure*?". In this subsection, we will discuss different models of defining the interference of a structure.

The interference model propose in [1] is based on the current network traffic. However, this model does require a priori information about the traffic in a network, which is often not available when designing the network topology due to the amount of the network traffic often is random and depends on the upper application layer. Thus, when we design a network topology to minimize the "interference", we prefer a static model of interference that is depending solely on a the distribution of the wireless nodes and, maybe, their transmission ranges.

Notice that, symmetric links are often preferred in wireless communications. In other words, a link uv exists in the communication graph if these two nodes u and v can communicate with each other directly, i.e., $|uv| \leq \min(r_u, r_v)$. Using this observation, Burkhart *et al.* [3] define the interference of a link uv as the number of nodes covered by two disks centered at u and v with radius $|uv|$. Let $D(u, r)$ denote the disk centered at node u with radius r. Specifically, they define the coverage of a link uv as

$$cov(uv) = \{w \mid w \; is \; covered \; by \; D(u, |uv|) \; or \; D(v, |uv|)\}.$$

Here, $cov(uv)$ represents the set of all nodes that could be affected by nodes u and v when they communicate with each other using the minimum power that can exactly reach each other only. We call this interference model as Interference based on Coverage model, and will use $IC(uv)$ to denote the interference of a link uv under this model. This model is chosen since whenever a link uv is used for a send-receive transaction all nodes whose distance to node u or node v is less than $\|uv\|$ will be affected.

The network is then represented by a geometric undirected *weighted* graph, $G = (V, E, W)$, with vertices representing wireless nodes, and edges representing communication links. The weight of each link uv is its interference number $IC(uv)$. See Figure 1 for an illustration. After assigning weights to all

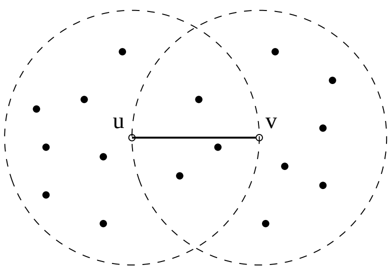

Fig. 1. The interference of link uv is the number of wireless nodes whose distance to node u or to node v is less than $\|uv\|$.

links, we call the graph the *interference graph*. Then, Burkhart *et al.* [3] proposed centralized methods to select a connected spanning subgraph of this interference graph while the maximum interference of selected links is minimized. They also proposed centralized and localized methods to select subgraphs with additional requirement that the subgraph is a Euclidean length spanner of the original communication graph.

Thus, given a subgraph H of the original communication graph G, the maximum interference, denoted as $MIC(H)$, of this structure H is defined as $\max_{e \in H} IC(e)$, and the total interference, denoted as $TIC(H)$, of this structure H is defined as $\sum_{e \in H} IC(e)$.

Notice that, in practice, the wireless devices often cannot adjust its transmission power to any number from 0 to its maximum transmission power. Usually, there are a sequence of discrete power levels that the wireless device can choose from. In this discrete power model, we clearly can extend the interference based on coverage model IC as follows. Given any link uv, let P_{uv} be the minimum power level such that nodes u and v can reach each other using this power, and let r_{uv} be the corresponding transmission range using the power P_{uv}, i.e. $P_{uv} = c_1 + c_2 \cdot r_{uv}^{\alpha}$. Then, the coverage of a link uv is defined as

$$cov(uv) = \{w \mid w \; is \; covered \; by \; D(u, r_{uv}) \; or \; D(v, r_{uv})\}.$$

And the interference $IC(uv)$ of a link uv is then the cardinality of $cov(uv)$. In the remainder of the paper, we will not distinguish this model from the model used [3]: we always use $IC(uv)$ to denote the interference of a link in both models.

Notice that the interference model used in [3] implicitly assume that the node u will send message to v and node v will send message to u at the same time. We argue that when u sends data to node v, typically node v only has to send a very short ack message to u. The communication then becomes one way by ignoring this small ack message from v. Clearly, when v is receiving message from node u, the nodes "nearby" node v cannot send any data, otherwise, the signal from u to v will be corrupted and thus interference occurs. Theoretically speaking, the transmission by another node w causes the interference to the transmission from node u to node v if the signal to interference and noise ratio (SINR) of the signal received by node v is below the threshold [3] of node v when node w transmits at a given power. To simplify the analysis of SINR, we assume that the transmission of a node w causes such interference if node v is within the transmission range of w. In other words, we say an interference occurs when v is within the transmission ranges of both node u and node w, and both node u and node w transmits signal to v. The number of such nodes w is the total number of nodes whose transmission will cause the interference of the signal received by node v. Considering such a node w, then the transmission of node w may cause interference to *all* nodes within its transmission range. Thus, to alleviate the interference, we would like to minimize the number of nodes within the transmission range of node w. We call such interference model as Interference based on Transmission model and will use $IT_H(w)$ to denote the interference of a node w under a given network topology H.

Thus, given a subgraph H of the original communication graph G, the transmission range of each node u is defined as $r_u = \max_{uv \in H} \|uv\|$. The interference number $IT_H(u)$ of a node u under Interference based on Transmission model is then defined as the cardinality of the set $\{v \mid \|uv\| \leq r_u\}$. The maximum interference of this structure H is defined as $\max_{u \in V} IT_H(u)$, and the total interference of this structure H is defined as $\sum_{u \in V} IT_H(u)$.

III. Link Based Interference

In this section, we design algorithms for topology control that minimize the maximum or the total interference of the resulting topology while preserving some properties of the network topology such as connectivity.

[3] The threshold of node v depends on the sensitivity of the antenna of node node v, the modulation technique of the signal, and other factors.

A. Minimize the Maximum Interference

Definition 1: The MIN-MAX link interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum interference $MIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Essentially, in [3], Burkhart *et al.* gave centralized methods for the MIN-MAX link interference with a property *connectivity*, and centralized and localized methods for the the MIN-MAX link interference with a property *bounded Euclidean spaning ratio*. In their algorithm (called LIFE) edges are sorted by their weights (interference) in ascending order. Starting from the edge with minimum weight, in each iteration of the algorithm an edge uv is processed. If nodes u and v are already connected, the edge uv is just ignored and otherwise it will be added to the topology. The algorithm continues till a connected graph is constructed. Clearly, the time complexity of this approach is $O(m \log m + hn)$, where h is the number of links in the final structure H. If a t-spanner structure is needed, they [3] add a link uv if the shortest path connecting u and v using previously added "short" links has length larger than t times the length of link uv; otherwise, link uv will not be added. Clearly, the time complexity of this approach is $O(m \log m + h(h + n \log n))$.

A graph property $\mathcal{P}$ is called *polynomially verifiable* if we can test whether any given graph H has this property $\mathcal{P}$ in time polynomial of the size of the graph H. For example, the connectivity property is polynomially verifiable, the bounded spanning ratio property is polynomially verifiable, the k-connectivity property is polynomially verifiable. Assume that we are given any *polynomially verifiable* property $\mathcal{P}$. The following binary search based approach is then straightforward.

Algorithm 1: Min-Max Link Interference with a given property $\mathcal{P}$

1) Sort the weight (i.e., interference number) of all links in ascending order. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link weights. Let $U = m$ and $L = 1$. Repeat the following steps until $U = L$.
2) Let $i = \lfloor \frac{L+U}{2} \rfloor$ and $w = w_i$.
3) Test if the structure H formed by all links with weight $\leq w$ has the property $\mathcal{P}$. If it does, then $U = i$, otherwise, then $L = i$.

Assume that the time complexity to test whether a given structure H (with n vertices and at most m links) has a property $\mathcal{P}$ takes time $\beta_{\mathcal{P}}(m, n)$. It is easy to show that the above binary search based approach has time complexity $O(m \log m + \beta_{\mathcal{P}}(m, n) \cdot \log n)$. For example, to test whether a structure is connected can be done in time $O(n)$, which implies that the min-max interference with connectivity can be done in time $O(m \log m + n \log n) = O(m \log n)$. Testing whether a given structure H is a t-spanner of the original graph G can be done in time $O(n(n \log n + m)) = O(n^2 \log n + mn)$, which implies that the min-max interference with *t-spanner* can be done in time $O(m \log m + n^2 \log^2 n + mn \log n) = O(n \log n(m + n \log n))$ using a binary search based approach described by Algorithm 1.

The following theorem is obvious.

Theorem 1: Algorithm 1 gives the optimum solution for MIN-MAX link interference problem with a given property $\mathcal{P}$.

B. Minimize the Total Interference

The maximum interference of the structure captures the worst link on the structure, however, it does not capture the overall performance of the structure in terms of the interference. In this section, we design algorithms that will minimize the *total* interferences of the structure while preserving some additional property $\mathcal{P}$.

Definition 2: The MIN-TOTAL link interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the total interference $TIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

When the given property $\mathcal{P}$ is just merely the connectivity of structure, to solve MIN-TOTAL link interference with a property $\mathcal{P}$ problem, it suffices to find the minimum spanning tree of the interference graph. The following lemma proves that the MST gives the optimum answer.

Lemma 2: MST gives the optimum solution for MIN-TOTAL link interference with *connectivity*.

PROOF. Assume the optimum graph, say G', is not MST and preserves connectivity. Since the total link interference in G' is equivalent to summation of link weights in G' it requires that G' has the minimum weight. Since G' is not MST, it must have weight less that MST which is impossible. ◻

Note that we will construct the minimum spanning tree of the interference graph, which is different from the Euclidean minimum spanning tree. Actually, the Euclidean MST (i.e. where the weight of each edge is the Euclidean length of the edge) can be $\Omega(n)$ times worse than the optimum for link interference MIN-TOTAL problem. The example illustrated by Figure 5 in [3] can be used to show that the Euclidean MST can be very bad for both link interference MIN-MAX and the link interference MIN-TOTAL problem. For that example, the maximum interference of the Euclidean MST is $O(n)$ and the total interference of the Euclidean MST is $O(n^2)$, while in the optimum structure, the maximum interference is $O(1)$, and the total interference is $O(n)$. Thus, Euclidean MST is $Omega(n)$ times worse than the optimum solution. Notice that $\Omega(n)$ is the worst possible ratio for any structure for both the MIN-MAX and the MIN-TOTAL link interference problem.

For the completeness of presentation, we review the example illustrated by Figure 4 here. The Euclidean MST for

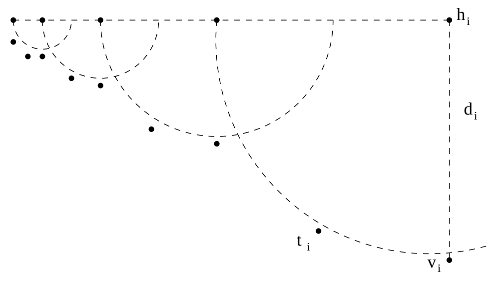

Fig. 2. Original graph with two node chains

this given network is shown in Figure 3. Figure 4 depicts the optimum solution for both link interference Min-Max problem and link interference Min-Total problem. Note that the same argument can be used for any topology containing the Nearest Neighbor Forest.

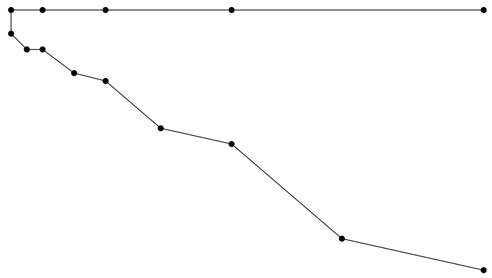

Fig. 3. Euclidean MST yields $\Omega(n)$ for Min-Max and $\Omega(n^2)$ for Min-Total.

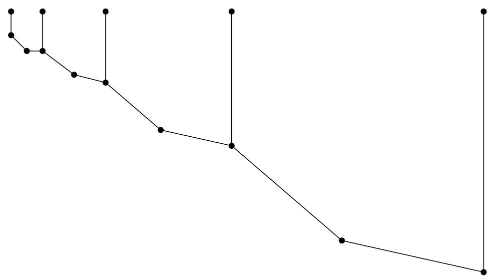

Fig. 4. Optimum solution yields constant value for Min-Max and $\Omega(n)$ for Min-Total.

Preserving the connectivity of the final structure H and minimizing the total interference can be optimally solved using the minimum spanning tree, it will be NP-hard to find the optimum structure when the property $\mathcal{P}$ is additive, e.g., being a t-spanner.

IV. Node Interference

In this section we define interference for each node instead of defining interference for each link. To study node interference problem we define two models. The first model is based on link interference and the second model is based on the number of nodes that are in the transmission region of a node.

A. Node Interference via Link

Given a network topology H, a node u will then only communicate using links in H. If node u communicate with a neighbor v with $uv \in H$, node u may experience the interference from $IC(uv)$ number of nodes. We then would like to know what is the worst interference number experienced by node u, i.e., we are then interested in $IC(u) = \max_{uv \in H} IC(uv)$. In this model the interference of each node u is interference of the link that has the maximum weight among all links that are connected to the node u.

Definition 3: Node Interference (Model 1): Given a structure H, the interference of a node u the maximum interference of all links incident on u, i.e.,

$$IC_H(u) = \max_{uv \in H} IC(uv).$$

Then the maximum node interference of a structure is then defined as $MNIC(H) = \max_{u \in V} IC_H(u)$, and the total node interference of a structure is then defined as $TNIC(H) = \sum_{u \in V} IC_H(u)$.

1) Minimize the Maximum Interference: First, we would like to minimize the maximum node interference.

Definition 4: The Min-Max node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Notice that the node interference of a node now does depend on the final topology, which introduces a level of difficulty compared with the link interference studied in subsection III-A. We first study how to find a connected topology whose maximum node interference is minimized. Surprisingly enough, we found that the minimum spanning tree based approach still produces the optimum network topology.

Algorithm 2: Min-Max Node Interference for connectivity

- Sort edges based on their weights (interference number) in ascending order. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link weights. Initialize structure H composed of node set V and empty links. Let $i = 1$. Repeat the following steps until a connected topology is found.
- Add edge uv with the weight w_i to the graph H if nodes u and v are not connected in H. Let $i = i + 1$.

Theorem 3: Algorithm 2 produces an optimum structure for Min-Max Node Interference for connectivity problem.

Proof. Let H be the final structure constructed by Algorithm 2. Assume the structure H is not optimum and OPT is an optimum structure. Consider the edge with the highest interference in H, say e. Then edge e doesn't belong to OPT (otherwise structure H would have been the optimum) and also the interference of all edges in OPT is less than the interference of edge e. This means a connected graph can be constructed with using edges whose interference is less than the interference of edge e. In that case our Algorithm 2 would have terminated before processing e and this is contradiction. □

2) Minimize the Total Interference: Similarly, we can also minimize the total node interference of the structure.

Definition 5: The Min-Total node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the total node interference $TNIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Solving the Min-Total node interference with a property $\mathcal{P}$ is not easy and since the simple form of this problem by requiring a connectivity property is similar to the min-total power symmetric connectivity, which is well-known to be NP-Hard. Thus, instead of trying to solving it optimally, we first give a good approximation algorithm to achieve the connectivity property. The following theorem proves that the MST (of the interference graph G) is a 2-approximation for the Min-Total node interference with connectivity.

Theorem 4: MST is a 2-approximation for the Min-Total node interference with connectivity problem.

PROOF. Consider any spanning tree T and let $I(T)$ denote the total node interference of graph T and let $W(T)$ denote the total weight of the links of graph T. Note that here the weight of each link is the interference of that link. Since the weight of each edge is assigned to at most two nodes, $I(T) \leq 2W(T)$. On the other hand, consider the spanning tree as a tree rooted at some nodes. For any leaf node u, the interference of the link that connects u to its parent is the interference that is assigned to node u; for any internal node v, the interference assigned to node u is less than or equal to the interference of the link between node v and its parent in the tree; and the interference assigned to root is some value greater than zero. Thus, the total interference of the nodes is greater than the total interference of the links and we have $W(T) < I(T)$. Now let OPT be the optimum structure. Clearly OPT is a spanning tree (i.e., cycles can be removed if there is any without increasing the total interference). We have $I(MST) \leq 2W(MST)$. Since MST is the minimum weight spanning tree, $W(MST) \leq W(OPT)$ and $W(OPT) < I(OPT)$. Consequently, $I(MST) < 2I(OPT)$. This finishes the proof. □

The MST based heuristics also works if the weight of each edge is some quality such as the power needed to support the the link, the delay of the link, or the SINR (Signal to Interference and Noise Ratio). Again, the Euclidean MST can be $\Omega(n)$ times worse than the optimum. Since the maximum interference is at most $O(n^2)$, obviously $\Omega(n)$ is the worst possible ratio.

B. Transmission based Interference

Notice that, when a topology H is used for routing, each wireless node typically adjusts its transmission power to the minimum that can reach its farthest neighbor in H. Considering this power level, we say that the the interference of each node u is the number of nodes inside its transmission range. Let r_u denote the transmission range of node u then the node interference is defined as follows:

Definition 6: NODE INTERFERENCE (MODEL 2): Given a structure H, the interference of a node u is number of nodes inside its transmission range, i.e.,

$$IT_H(u) := |\{v \mid \|uv\| \leq r_u\}|.$$

Here $r_u = \max_{uv \in H} \|uv\|$.

Then similarly the maximum node interference of a structure is then defined as $MNIT(H) = \max_{u \in V} IT_H(u)$, and the total node interference of a structure is then defined as $TNIT(H) = \sum_{u \in V} IT_H(u)$.

1) Minimize the Maximum Interference: First, we would like to minimize the maximum node interference.

Definition 7: The MIN-MAX node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIT(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Consider node u and let $N(u)$ be the number of neighbors of node u when node u adjusts its transmission range to maximum. Node u can adjust its transmission range to have exactly k neighbors ($0 \leq k \leq N(u)$). In other words, each node u can set its interference to any value between 0 and $N(u)$ by using the appropriate transmission range. Having this property, solving the MIN-MAX node interference with a property $\mathcal{P}$ problem is only a simple binary search.

Algorithm 3: MIN-MAX Node Interference with a property $\mathcal{P}$ for model 2.

1) Let $U = n - 1$ and $L = 1$. Repeat the following steps until $U = L$.
2) Let $i = \lfloor \frac{L+U}{2} \rfloor$ and let H_i be the graph formed by connecting each node u to its first i-shortest links. Notice that, if u has less than i neighbors in the original graph, then u will only connect to all its $N(u)$ neighbors.
3) Test if the structure H_i has the property $\mathcal{P}$. If it does, then $U = i$, otherwise, then $L = i$.

Assume Algorithm 3 gives an interference value i. Since setting the interference of each node to a value less than i cannot preserve the property $\mathcal{P}$. The following theorem is then obvious.

Theorem 5: Algorithm 3 produces the optimum solution for the MIN-MAX Node Interference with a property $\mathcal{P}$.

2) Minimize the Total Interference: Similarly, we can also minimize the total node interference of the structure.

Definition 8: The MIN-TOTAL node interference with a property $\mathcal{P}$ problem is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the total node interference $TNIT(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Solving the MIN-TOTAL node interference problem for Model 2 is not easy and it seems to be NP-Hard to find the optimum answer. Here we give an efficient heuristics to find a structure that is practically good.

We construct a directed graph $G' = (V', E', W')$ as follows: for each edge uv of G, we introduce two additional vertices $[uv]$ and $[vu]$. Each node u, sorts its neighbors $v_1, v_2, ..., v_k$ in ascending order of distances from u. Then we connect node u to node $[uv_1]$ using directed link $u[uv_1]$ and we assign weight 1 to it; we also define a directed link $[uv_1]u$ and we assign weight 0 to link $[uv_1]u$. We also connect vertices $[uv_i]$ and $[uv_{i+1}]$ using two directed links $[uv_i][uv_{i+1}]$ and $[uv_{i+1}][uv_i]$ ($1 \leq i < k$) and assign weight 1 to all those links $[uv_i][uv_{i+1}]$ and we assign weight 0 to all links $[uv_{i+1}][uv_i]$ ($1 \leq i < k$). All pairs $[uv], [vu]$ are connected also. Assume node u is the p^{th} nearest neighbor of node v and node v is the q^{th} nearest neighbor of node u. Then we assign weight p to the edge $[uv][vu]$ and weight q to $[vu][uv]$. See Figure 5 and Figure 6 for an illustration. Figure 5 depicts the original graph and Figure 6 shows the transformed graph. All dashed edges have weight 0. Now we start from any node $u \in V$ and we solve the min-cost multicast problem to all other nodes $v \in V$. It is easy to show that the min-cost multicast problem in G' is equal to the min-total node interference graph in G.

We then introduce a greedy based algorithm for this multicast problem in the directed graph G'. The algorithm starts with and empty set of *processed nodes*, denoted by A, and picks a random node u and puts it in the set A. We define the distance between a node v that does not belong to set A and set A as the shortest path starting from a node in set A to v. Then in each iteration the node that is the closest to the set A is added

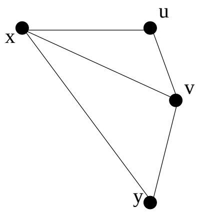

Fig. 5. The original communication graph.

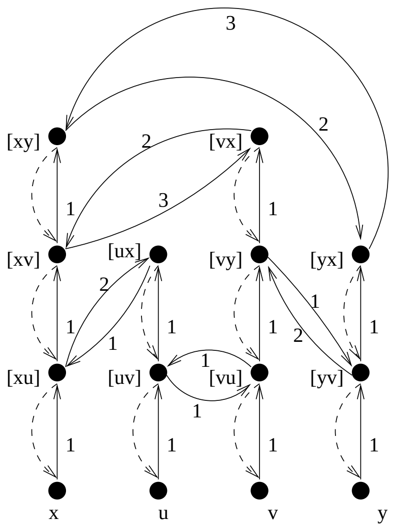

Fig. 6. The transformed graph.

to set A and the distances of nodes to the set A is updated. The algorithm continues till all desired nodes are in A. Let H_u be the final structure constructed when node u is first put to the set A.

To find the best structure possible, we will construct the structures H_{v_i} for all nodes $v_i \in V$ and then finds the structure with the minimum total nodal interference.

V. Conclusion and Future Work

Topology control draw considerable attentions recently in wireless ad hoc networks for energy conservation. In this paper, we studied various problems of topology control when we have to minimize the interference of the constructed structure. We optimally solve some problems, give approximation algorithms for some NP-hard questions, and also give some efficient heuristics for some questions that seems to be NP-hard. We are conduct simulations to see how these new structures perform for random wireless networks. We would like to know whether our greedy heuristics for the min-total node interference does give a constant approximation guarantee.

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