Stretch Factor of Curveball Routing in Wireless Network: Cost of Load Balancing

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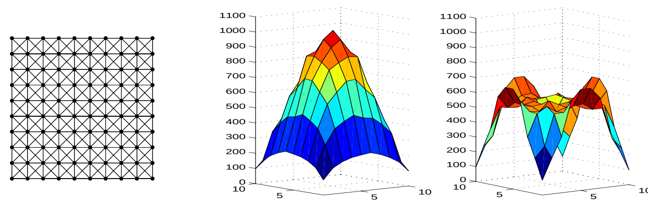
***Abstract*— Routing in wireless networks has been heavily studied in the last decade and numerous routing protocols were proposed in literature. Most of the existing routing protocols are based on shortest path routing. Shortest path routing enjoys minimizing the total delay, but may lead uneven distribution of traffic load in a network. For example, wireless nodes in the center of a network usually have heavier traffic load since most of the shortest routes go through the center. To solve this problem, Popa *et al.* [1] recently proposed a novel routing method, called *curveball routing* (CBR), which can balance the traffic load and vanish the crowded center effect. In CBR, nodes are mapped on a sphere and packets are routed on those virtual coordinates on the sphere. While CBR achieves better load balancing for the network, it also uses longer routes than the shortest paths. This can be treated as the cost of load balancing. In this paper, we focus on studying this cost of load balancing for curveball routing. Specifically, we theoretically prove that for any network, the distance traveled by the packets using CBR is no more than a small *constant* factor of the minimum (the distance of the shortest path). The constant factor, we called stretch factor, is only depended on the ratio between the size of the network and the radius of the sphere used in CBR. We then conduct extensive simulations to evaluate the stretch factor and load distribution of CBR and compare them with the shortest path routing in both grid and random networks. We also study the trade-off between stretch factor and load balancing.**

I. Introduction

Routing is one of the key topics in wireless networks and various routing protocols have been proposed and studied. Most of the routing protocols are based on shortest path algorithms where the packets are traveled via the shortest path between a source and a destination. Taking the shortest path can achieve smaller delay and shorter traveled distance, however it may also lead uneven distribution of traffic load in a network. As observed in previous work [2], [3], under uniform communication scenario, the center of the network becomes "crowded", since more shortest paths go through the center than through the periphery of the network. This is also the observation from the metropolitan transportation system where the downtown area is always the "hot spot" for traffic congestion. Fig. 1 shows a simple simulation result of this scenario. The network is distributed on a 10×10 grid as shown in Figure 1(a). Consider a uniform communication scenario where each node sends one packet to all other nodes using the shortest path routing (SPR). Fig. 1(b) illustrates the cumulative node traffic (*i.e.*, number of packets passing through) for each node. It is clear that nodes in the center area have the most traffic load, thus they may run out of their batteries more quickly than nodes in the periphery.



(a) topology (b) load of SPR (c) load of CBR

Fig. 1. Crowded center effect: nodes in the center area have much heavier traffic load than nodes in the periphery. The curveball routing can eliminate crowded center effect by spreading the traffic on the sphere.

To address the uneven load distribution problem, people investigate on load balancing routing for large wireless networks. By spreading the traffic across the wireless network via the elaborate design of the routing algorithm, load balancing routing averages the energy consumption. This extends the lifespan of the whole network by extending the time until the first node is out of energy. Load balancing is also useful for reducing congestion hot spots thus reducing wireless collisions. There are already several load balancing routing protocols [4]–[6] in literature. However, most of them try to dynamically adjust the routes to balance the real time traffic load based on the knowledge of current load distribution (or current remaining energy distribution), which is not very scalable for large wireless networks. Multi-path routing [2] was also used for load balancing. However, [7] showed unless using a very large number of paths the load distribution is almost the same as single path routing. Hyytia and Virtamo [3] also studied how to avoid the crowded center problem by analyzing the load probability in a dense network. They proposed a randomized choice between shortest path and routing on inner/outer radii to level the load.

Recently, Popa *et al.* [1] proposed a novel load balancing routing method, called *curveball routing* (CBR). In curveball routing, nodes are mapped on a sphere and packets are routed on those virtual coordinates. Instead of using the Euclidean distance as the routing metric, CBR uses the spherical distance between the virtual coordinates. Fig. 1(c) shows the load distribution of CBR in the same grid network under uniform communication scenario. It is clear that CBR can vanish the

This work was supported in part by the US National Science Foundation (NSF) under Grant No. CNS-0721666 and funds provided by the University of North Carolina at Charlotte.

crowded center effect. In [1], CBR is evaluated in both ns2 simulator and a large testbed in Berkeley. All simulations confirm that CBR can achieve better load balancing.

However, there's no such thing as a free lunch. While CBR achieves better load balancing, it also uses longer routes than the shortest paths. In general, this means CBR may need more relaying nodes to deliver the packets thus leads to large energy consumption. We treat the increase of path length as the cost of load balancing for CBR. In [1], the authors did not provide any formal study on this cost, except claimed that "*In the presented simulation, curveball routing increases the average path length by less than* 7.5% *compared to the greedy paths. Similarly, the longest path increases by* 59%".

In this paper, we focus on theoretically studying the cost of load balancing for CBR. We formally define the *competitiveness* and *stretch factor* of any routing method compared to SPR. Given a routing method $\mathcal{A}$, let $\mathbf{P}_{\mathcal{A}}(\mathbf{s},\mathbf{t})$ be the path found by $\mathcal{A}$ to connect the source node $\mathbf{s}$ and the target node $\mathbf{t}$. A routing method $\mathcal{A}$ is called *α-competitive* if for every pair of nodes $\mathbf{s}$ and $\mathbf{t}$, the total length of path $\mathbf{P}_{\mathcal{A}}(\mathbf{s},\mathbf{t})$ is within a constant factor α of the length of the shortest path connecting $\mathbf{s}$ and $\mathbf{t}$ in the network. The constant factor α is called *stretch factor* of $\mathcal{A}$. Then, we theoretically prove that for any networks, the stretch factor of CBR is bounded by $max(\frac{\pi}{2}(1+\epsilon),\pi)$, where ϵ is a constant parameter only depends on the ratio between the size of the network and the radius of the sphere used in CBR. In other words, CBR can guarantee the total distance traveled by packets is constant competitive even in the worst case.

II. Curveball Routing

The basic idea of curveball routing [1] is mapping all nodes in a 2D network onto a 3D sphere. Since the surface of the sphere is symmetric, if nodes only communicate on the surface and communication is uniform, there will be no crowded center effect. One-to-one mapping between nodes in a plane and nodes on a sphere has been well-studied in projective geometry. A simple *stereographic projection* [8] can map an infinite plane onto a sphere and vice versa. Fig. 2(a) illustrates the mapping method used by CBR, which is a stereographic projection. For a wireless network, the area in which the wireless nodes lie corresponds to a finite region in the plane. Let this region be $\mathbb{P}$. With the information of the network region, we can place a sphere $\mathbb{S}$ centered at the center $O(0,0,0)$ of the network. The radius R of $\mathbb{S}$ is an adjustable parameter for CBR. Any point $m(x,y,0)$ in $\mathbb{P}$ maps to its projection $m'(x',y',z')$ on the sphere $\mathbb{S}$, which is the intersection of $\mathbb{S}$ and the line through m and the north pole ($N(0,0,R)$). As shown in Fig. 2(a), the projection m' of a node m inside the equator C is on the southern hemisphere, while the projection n' of a node n outside C is on the northern hemisphere. Note that stereographic projection preserves circles perfectly. That is, a circle on the sphere maps to a circle in the plane and vice versa.

After mapping the nodes on the sphere, CBR routes packets on the spherical shortest paths on the sphere instead of the Euclidean shortest paths in the 2D plane. Note that the mapping preserves topological neighborhoods, it only changes the cost of the links. For any existing link mn between two nodes m and n in the network, CBR uses the shortest distance on the sphere between their projected nodes m' and n' (denoted by $d(m'n')$) as the cost of link mn. Then, routing decision can be made either by shortest path algorithm or greedy algorithm on the new costs. Hereafter, we assume the shortest path algorithm is used, *i.e.*, CBR chooses the route with smallest total spherical distance. CBR is easy to be implemented by the simple modification of shortest path routing or greedy routing and has negligible additional computational overhead. The only introduced overhead is that each node needs to compute its neighbors' spherical coordinates.

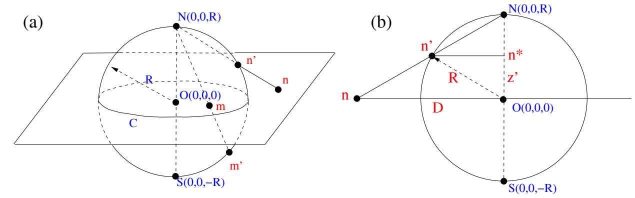


Fig. 2. Projecting the network on a sphere.

III. Stretch Factor of Curveball Routing

In this section, we study the stretch factor of CBR, *i.e.*, the ratio between the total length of the path taken by CBR and the length of the shortest path. Before giving the proof, we need to present some preliminaries for stereographic projection.

Lemma 1: Assume that the furthest wireless node is of distance D from the center, then the z' value of the highest projection on the sphere (denoted this value as k) is $\frac{\epsilon-1}{\epsilon+1}R$, where $\epsilon = D^2/R^2$.

Proof: First, we assume $D \geq R$. Let n be the furthest node, n' be its projection, and n^* be the projection of n' on line segment NS, as shown in Fig. 2(b). Then $k = z'_{max} = R\sin\angle n^*n'O$. Note that $\angle n^*n'O = \frac{\pi}{2} - \angle n'ON = \frac{\pi}{2} - (\pi - 2\angle nNO) = 2\angle nNO - \frac{\pi}{2}$. Thus, $k = R\sin(2\angle nNO - \frac{\pi}{2}) = R(\sin^2\angle nNO - \cos^2\angle nNO) = R(\frac{D^2}{R^2+D^2} - \frac{R^2}{R^2+D^2}) = R\frac{D^2-R^2}{D^2+R^2}$. When we choose $R = \frac{D}{\sqrt{\epsilon}}$ (*i.e.*, $\epsilon = D^2/R^2$), thus $k = \frac{\epsilon-1}{\epsilon+1}R$. If $D < R$, we can draw the same conclusion through a similar proof. ∎

Recall that circles on the sphere map to circles in the plane in stereographic projection, thus the projection of a great circle on the sphere $\mathbb{S}$ is also a circle in the plane. Let $d(m'n')$ be the length of an arc $C_{m'n'}$ from a projection m' to a projection n' along a great circle on the surface of $\mathbb{S}$, $d(mn)$ be the length of the arc C_{mn} between m and n along the projection of the great circle in the plane, and $||mn||$ be the Euclidean distance between m and n in the plane. The next two lemmas show that $d(m'n')$ is bounded from above and below by the Euclidean distance $||mn||$ in the plane.

Lemma 2: Consider any two nodes m and n in the plane with their projections m' and n' on the sphere $\mathbb{S}$, we have

$$||mn|| \leq \beta_1 d(m'n'),$$

where $\beta_1 = 1$ when both two nodes are inside the equator C and $\beta_1 = \frac{1+\epsilon}{2}$ otherwise.

Proof: First, since the Euclidean distance of two points is always smaller than the distance along any arc passing them, $||mn|| \leq d(mn)$. Thus, we only need to prove $d(mn) \leq \beta_1 d(m'n')$. Let us consider the following three cases.

Case 1: both m and n are outside C. See Fig. 3(a) for an illustration. It is a one-to-one mapping between one point on arc $C_{m'n'}$ and one point on arc C_{mn}. $\int_{C_{m'n'}} dx' = d(m'n')$, where dx' is a miniature segment on $C_{m'n'}$. Similarly, $\int_{C_{mn}} dx = d(mn)$, where dx is the projection of dx' in the plane. $p'q'$ is a tiny segment on $C_{m'n'}$ with its length $dx' \to 0$, and $dx' = ||p'q'||$. The projection of $p'q'$ is pq with the length $dx = ||pq||$. Denote p^* as the projection of p' on line segment NS. The z' value of p^* (or p') is denoted by z_{p^*}. Then

$$\frac{||Np^*||}{||NO||} = \frac{R - z_{p^*}}{R}.$$

When $dx', dx \to 0$, *i.e.*, $pq, p'q' \to 0$, pq and $p'q'$ are in the same plane (the plane defined by nodes N,p and q). Then, due to the similarity of triangles $\triangle Np'p^*$ and $\triangle NpO$,

$$\frac{dx'}{dx} = \frac{||p'q'||}{||pq||} = \frac{||Np'||}{||Np||} = \frac{||Np^*||}{||NO||} = \frac{R - z_{p^*}}{R}.$$

Because the highest value of z_{p^*} is k from Lemma 1, we have

$$\frac{dx'}{dx} \geq \frac{R-k}{R} = \frac{R - \frac{\epsilon-1}{\epsilon+1}R}{R} = \frac{2}{1+\epsilon}.$$

Thus,

$$d(mn) = \int_{C_{mn}} dx \leq \int_{C_{m'n'}} \frac{1+\epsilon}{2} dx' = \frac{1+\epsilon}{2} d(m'n').$$

Case 2: both m and n are inside C. See Fig. 3(b) for an illustration. Similar to Case 1, we can define dx and dx'. Now, since node p^* is always below the plane,

$$\frac{dx'}{dx} = \frac{||p'q'||}{||pq||} = \frac{||Np'||}{||Np||} = \frac{||Np^*||}{||NO||} \geq 1.$$

Thus,

$$d(mn) = \int_{C_{mn}} dx \leq \int_{C_{m'n'}} dx' = d(m'n').$$

Case 3: one of m and n is inside C and the other is outside C. Without loss of generality, we assume m is the one outside C. Then arc $C_{m'n'}$ intersects arc C_{mn} at node $l(l')$ which is also on the equator C. By dividing the arc $C_{m'n'}$ into two parts $C_{m'l'}$ and $C_{l'n'}$, we can apply results from Case 1 and Case 2 to them. Thus, we have

$$\begin{aligned} d(mn) &= d(ml) + d(ln) \leq \frac{1+\epsilon}{2} d(m'l') + d(l'n') \\ &\leq max(\frac{1+\epsilon}{2}, 1)(d(m'l') + d(l'n')) \\ &= max(\frac{1+\epsilon}{2}, 1) d(m'n'). \end{aligned}$$

Note that since there is at least one node outside C, $D > R$, *i.e.*, $\epsilon > 1$. Thus $d(mn) \leq \frac{1+\epsilon}{2} d(m'n')$. ■

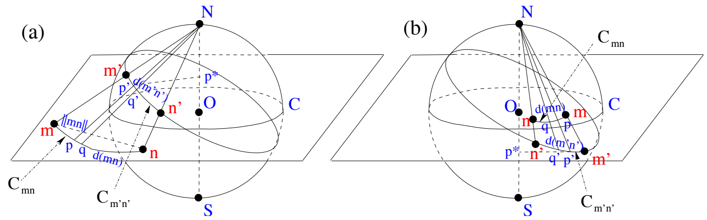


Fig. 3. The length of the projection $d(mn)$ is bounded by the length of the segment of great circle $d(m'n')$ on the sphere. $d(mn) \leq \beta_1 d(m'n')$.

Lemma 3: Consider any two nodes m and n in the plane with their projections m' and n' on the sphere $\mathbb{S}$, we have

$$d(m'n') \leq \beta_2 ||mn||,$$

where $\beta_2 = \frac{\pi}{2}$ when both two nodes are outside the equator C and $\beta_2 = \pi$ otherwise.

Proof: Since arc C_{mn} is a segment between m and n on a circle (which may not be centered at O), we have $d(mn) \leq \frac{\pi}{2}||mn||$ (Note that when C_{mn} is a half circle, the ratio of $d(mn)$ to $||mn||$ reaches its maximum of $\frac{\pi}{2}$). Thus, we only need to prove that $d(m'n')$ is less or equal to a constant time of $d(mn)$. Similar to the proof of Lemma 2, assume that dx' is a miniature segment on $C_{m'n'}$ and dx is the projection of dx' in the plane.

Case 1: both m and n are outside C. From the proof of Case 1 in Lemma 2, we know $\frac{dx'}{dx} = \frac{R - z_{p^*}}{R} \leq 1$. Thus, $dx' \leq dx$, and

$$d(m'n') = \int_{C_{m'n'}} dx' \leq \int_{C_{mn}} dx = d(mn).$$

Case 2: both m and n are inside C. From the proof of Case 2 in Lemma 2, we know $\frac{dx'}{dx} = \frac{||Np^*||}{||NO||} \leq 2$ (when $p^* = S$, this ratio reaches maximum). Thus, $dx' \leq 2dx$, and

$$d(m'n') = \int_{C_{m'n'}} dx' \leq \int_{C_{mn}} 2dx = 2d(mn).$$

Case 3: one of m and n is inside C and the other is outside C. Again let $l(l')$ be the intersection point on C and assume m is the one outside, we have

$$\begin{aligned} d(m'n') &= d(m'l') + d(l'n') \leq d(ml) + 2d(ln) \\ &\leq 2(d(ml) + d(ln)) = 2d(mn). \end{aligned}$$

This concludes the proof. ■

Now we are ready to prove the main theorem of this paper about the stretch factor of curveball routing. Recall that the stretch factor of a routing method $\mathcal{A}$ is a constant α, if and only if for every pair of nodes $\mathbf{s}$ and $\mathbf{t}$, the total length of path $\mathbf{P}_{\mathcal{A}}(\mathbf{s}, \mathbf{t})$ found by $\mathcal{A}$ is within α times of the length of the shortest path connecting $\mathbf{s}$ and $\mathbf{t}$ in the network. In other words, here we want to prove that CBR can find a path whose length is within a small constant factor of the minimum even in the worst case scenario. There are four paths we will use in the proof: (1) $\mathbf{P}_{\mathcal{SPR}}(\mathbf{s}, \mathbf{t})$ is the shortest path between the source $\mathbf{s}$ and the destination $\mathbf{t}$ in the 2D plane; (2) $\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s}, \mathbf{t})$ is the surface path connecting all the projections on the sphere of each node along $\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})$ using the spherical distance; (3) $\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})$ is the path found by CBR protocol in the 2D plane; and (4) $\mathbf{P}'_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})$ is the surface path on the sphere connecting all the projections of each node along $\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})$. Note that in any two points along a path in the 2D plane, the shortest distance is the length of the straight line connecting them, meanwhile the shortest spherical distance of its projection on the sphere is the length of a segment (an arc) of a great circle. For a path $\mathbf{P}_{\mathcal{A}}$ in the plane, we define $||\mathbf{P}_{\mathcal{A}}||$ as the summation of the Euclidian distance of each link in $\mathbf{P}_{\mathcal{A}}$. For a path $\mathbf{P}'_{\mathcal{A}}$ on the sphere, we define $||\mathbf{P}'_{\mathcal{A}}||$ as the summation of the length of each arc in $\mathbf{P}'_{\mathcal{A}}$.

Theorem 4: The stretch factor of CBR routing is bounded by β_3, *i.e.*,

$$||\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq \beta_3 ||\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||,$$

where $\beta_3 = \frac{\pi}{2}(1+\epsilon)$ when $\epsilon \geq 1$ and $\beta_3 = \pi$ otherwise. *I.e.*, $\beta_3 = max(\frac{\pi}{2}(1+\epsilon), \pi)$.

Proof: Let $\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t}) = v_0, v_1, v_2, \cdots, v_n$, where $v_0 = \mathbf{s}$ and $v_n = \mathbf{t}$. Let the projection of $\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})$ on the sphere $\mathbf{P}'_{\mathcal{CBR}}(\mathbf{s},\mathbf{t}) = v'_0, v'_1, v'_2, \cdots, v'_n$. Similarly, let $\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t}) = u_0, u_1, u_2, \cdots, u_m$, where $u_0 = \mathbf{s} = v_0$ and $u_m = \mathbf{t} = v_n$. Let the projection of $\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})$ on the sphere $\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t}) = u'_0, u'_1, u'_2, \cdots, u'_m$, where $u'_0 = \mathbf{s}' = v'_0$ and $u'_m = \mathbf{t}' = v'_n$. Note that m may not equal to n.

Case 1: $\epsilon < 1$. *I.e.*, $D < R$ and all nodes are inside the equator C. From Lemma 2, we know $||v_{i-1}v_i|| \leq d(v'_{i-1}v'_i)$, therefore, $||\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| = \sum_{i=1}^{n} ||v_{i-1}v_i|| \leq \sum_{i=1}^{n} d(v'_{i-1}v'_i) = ||\mathbf{P}'_{CBR}(\mathbf{s},\mathbf{t})||$. According to the CBR protocol, $||\mathbf{P}'_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq ||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$ since $\mathbf{P}'_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})$ has the shortest total spherical distance among all routes on the sphere surface connecting $\mathbf{s}'$ and $\mathbf{t}'$. From Lemma 3, we have $d(u'_{i-1}u'_i) \leq \pi ||u_{i-1}u_i||$. Thus, $||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})|| = \sum_{i=1}^{n} d(u'_{i-1}u'_i) \leq \sum_{i=1}^{n} \pi ||u_{i-1}u_i|| = \pi ||\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$. Consequently, $||\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq ||\mathbf{P}'_{CBR}(\mathbf{s},\mathbf{t})|| \leq ||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})|| \leq \pi ||\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$.

Case 2: $\epsilon \geq 1$. *I.e.*, $D \geq R$ and some nodes are outside the equator C. From Lemma 2, we know $||v_{i-1}v_i|| \leq \max(\frac{1+\epsilon}{2}, 1) d(v'_{i-1}v'_i) = \frac{1+\epsilon}{2} d(v'_{i-1}v'_i)$, since $\frac{(1+\epsilon)}{2} > 1$. Thus, $||\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq \frac{1+\epsilon}{2} ||\mathbf{P}'_{CBR}(\mathbf{s},\mathbf{t})||$. According to the CBR protocol, $||\mathbf{P}'_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq ||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$. From Lemma 3, we have $d(u'_{i-1}u'_i) \leq max(\frac{\pi}{2}, \pi) ||u_{i-1}u_i|| = \pi ||u_{i-1}u_i||$. Thus, $||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})|| \leq \pi ||\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$. Consequently, $||\mathbf{P}_{\mathcal{CBR}}(\mathbf{s},\mathbf{t})|| \leq \frac{1+\epsilon}{2} ||\mathbf{P}'_{CBR}(\mathbf{s},\mathbf{t})|| \leq \frac{1+\epsilon}{2} ||\mathbf{P}'_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})|| \leq \frac{\pi}{2}(1+\epsilon) ||\mathbf{P}_{\mathcal{SPR}}(\mathbf{s},\mathbf{t})||$. ■

Theorem 4 gives a theoretical bound of the stretch factor of CBR protocol. It shows that the path length in CBR protocol is not too much different from the shortest path routing. The stretch factor is only related to $\epsilon = \frac{D^2}{R^2}$. Recall that D is the distance of the furthest node to the center of the network and R is the radius of the projection sphere. Given a network with fixed D, we can control R to adjust the stretch factor.

IV. SIMULATION

We now evaluate the performance of CBR via extensive simulations for both grid networks and random networks by our own developed simulator. In both cases, wireless nodes are distributed in a 20×20 square area and a simple unit disk graph propagation model is used. In CBR, the center of the sphere $\mathbb{S}$ is located at the center of the deployment area. The virtual coordinates of all projection nodes on the sphere are then generated. The performance of CBR is measured in terms of traffic load and stretch factor. It is clear that the size of the sphere affects the distribution of the mapped nodes on the sphere, thus affects the performance. In our simulations, we try different radii of the sphere (*i.e.*, various ratio of $\frac{D}{R}$.) and plot the performance figures with different ratio of $\frac{D}{R}$.

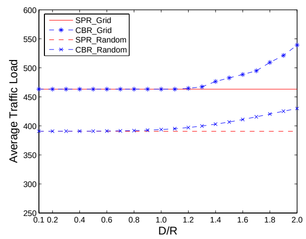
Grid Networks: We first deploy the 100 nodes on a 10×10 grid inside the 20×20 square area, then set the transmission range of all nodes to 3. The resulted topology is shown in Fig. 1(a). We compare the performance of the shortest path routing (SPR) and the curveball routing (CBR) under the uniform communication scenario where every pair of nodes has unit message to communicate.

Fig. 4(a-c) show the average, maximum, and standard deviation (STD) of traffic load for all nodes in the network for SPR and CBR with different radii. It is clear that the maximum load (or STD of load) of CBR is smaller than SPT when $\frac{D}{R} < 1.9$. This shows the CBR indeed balances load among all nodes. The minimum values occur when $\frac{D}{R}$ equals to 1.4 or 1.5 (For example, the maximum load can be reduced about 40% by CBR compared to SPT at $\frac{D}{R} = 1.4$).

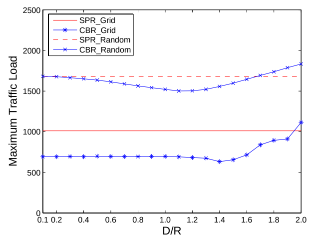
As the cost of load balancing, we also study the stretch factor (SF) of CBR. From Theorem 4, the SF of CSR is bounded by $\frac{\pi}{2}(1+\frac{D^2}{R^2})$ when $\frac{D}{R} \geq 1$ and π when $\frac{D}{R} < 1$. We measure the SF for each route generated by CBR in our simulation. Fig. 4(d) shows the average and maximum stretch factor of CBR on all routes. The simulation results of SFs confirm our theoretical bounds. Actually the practical SFs are much smaller than the bounds. The average SFs are all very close to 1, *i.e.*, the distance traveled by the packets in CBR is almost the same as the minimum (the distance of the shortest path). The maximum SFs of CBR increase with the value of $\frac{D}{R}$ increases which confirms our theoretical proof. Therefore, given a network, the cost of load balancing increases with the decrease of the radius of the sphere. There is a clear tradeoff between load balancing and stretch factor.

We also studied the impact of the number of nodes. We deploy the 400 nodes on a 20×20 grid inside the 20×20 square area and set the transmission range of all nodes to 1.5. Fig. 4(e-h) show the simulation results which basically have the same pattern as of the results with 100-nodes network. The minimum values of maximum load (Fig. 4(f)) or STD of load (Fig. 4(g)) occur when $\frac{D}{R}$ equals to 1.3 or 1.4.

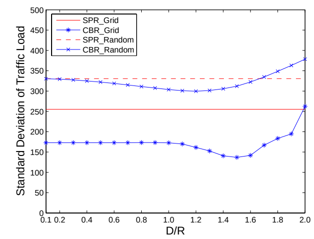
Random Networks: We test the performance of CSR with random networks with both 100 nodes and 400 nodes. Transmission range is set to 4 (for 100-nodes network) or 2 (for 400-nodes network). In each case, we generate 100 random networks and take the average for all results. The results are also plotted in Fig. 4(a-d) and Fig. 4(e-h). All the patterns of performances are still almost the same as of grid networks. But the minimum values of maximum load (Fig. 4(b)(f)) or STD



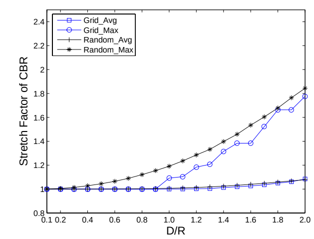
(a) Average Traffic Load



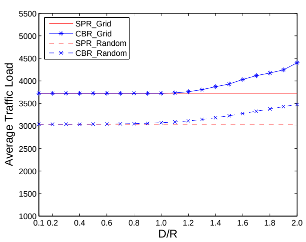
(b) Maximum Traffic Load



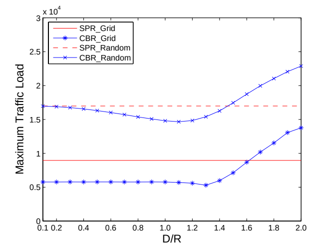
(c) STD of Load



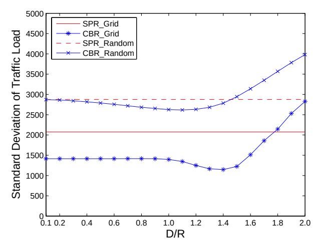
(d) Stretch Factor of CBR



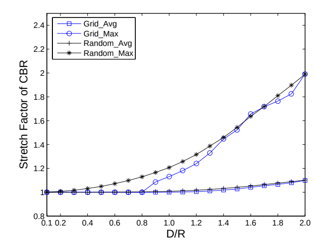
(e) Average Traffic Load



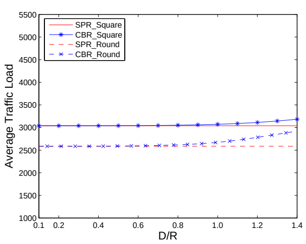
(f) Maximum Traffic Load



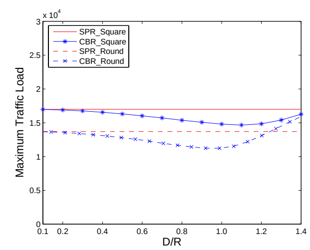
(g) STD of Load



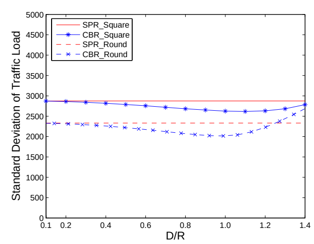
(h) Stretch Factor of CBR



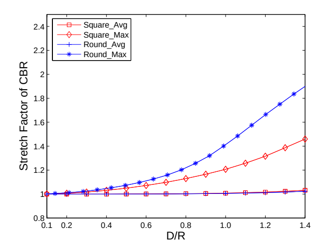
(i) Average Traffic Load



(j) Maximum Traffic Load



(k) STD of Load



(l) Stretch Factor of CBR

Fig. 4. Average, maximum, STD of load, and stretch factor of CBR compared with SPR. (a-d) are for 100-nodes grid and random networks in a square area, (e-h) are for 400-nodes grid and random networks in a square area, and (i-l) are for 400-nodes random networks in either a square area or a disk area.

of load (Fig. 4(c)(g)) occur when $\frac{D}{R}$ equals to 1.0 or 1.1. It is interesting that random networks have smaller average load than grid networks. Remember that the transmission range in random networks is larger than the one in grid networks (in order to guarantee the network connectivity). Thus, average hop count of routes in random networks is less than the one in grid networks which leads to lower total load in random networks. On the other hand, random networks have larger maximum load than grid networks. This is due to more uneven distribution of nodes in random networks than perfect grid networks. This also causes that random networks have larger stretch factors than grid networks.

We also studied the impact of the shape of the network. Instead of a 20×20 square area, we deploy the nodes randomly in a disk area with radius equal to 10. Fig. 4(i-l) demonstrates the results in both square and round networks with 400 nodes. The similar conclusions can be drawn from these figures.

V. Conclusion

Popa *et al.* [1] recently proposed curveball routing (CBR), which can balance the traffic load and vanish the crowded center effect under uniform communication scenario. In this paper, we studied the stretch factor of CBR. We theoretically proved that for any network the distance traveled by the packets using CBR is no more than a small constant factor $max(\frac{\pi}{2}(1+\epsilon), \pi)$ of the minimum (the distance of the shortest path). The stretch factor is only depended on $\epsilon = \frac{D^2}{R^2}$, where $\frac{D}{R}$ is the ratio of the size of the network to the radius of the sphere used in CBR. Via simulation, we showed that there is a trade-off between balancing the traffic load and reducing the stretch factor, *i.e.*, appropriate radius of the sphere should be selected to optimize the load distribution without increasing stretch factor too much. We leave further theoretical study of load distribution of CBR as our future work.

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