Localized Topologies with Bounded Node Degree for Three Dimensional Wireless Sensor Networks

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***Abstract*—Three dimensional (3D) wireless sensor networks have attracted a lot of attention due to its great potential usages in both commercial and civilian applications. Topology control in 3D sensor networks has been studied recently. Different 3D geometric topologies were proposed to be the underlying network topologies to achieve the sparseness of the communication networks. However, most of the proposed 3D topologies cannot bound the node degree, i.e., some nodes may need to maintain large number of neighbors in the constructed topologies, which is not energy efficient and may lead to large interference. In this paper, we extend several existing 3D topologies to a set of new 3D topologies with bounded node degree. We provide theoretical analysis on their power efficiency and node degree and also simulation evaluations over random 3D sensor networks. The simulation results confirm nice performance of these proposed 3D topologies.**

***Keywords*-Topology control, power efficiency, power spanner, node degree, three dimensional wireless sensor networks.**

I. Introduction

Due to its wide-range potential applications (such as environmental data collection, pollution monitoring, space exploration, disaster prevention and tactical surveillance), 3D wireless sensor network has recently emerged as a premier research topic. Most current research in 3D sensor networks primarily focuses on coverage [1], [2], connectivity [2], [3], and routing issues [4], [5]. In this paper, we focus on how to efficiently control the 3D network topology to maintain both network connectivity and energy efficiency of routes in 3D wireless sensor networks.

Topology control have been well-studied for two dimensional (2D) wireless ad hoc and sensor networks in the past decade [6]–[12]. Topology control methods allow each sensor node *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a structure that can support energy efficient routing and improve the overall network performance. Given the dynamic nature of sensor networks, the topology should be locally and self-adaptively maintained without affecting the whole network and the communication cost during maintaining should not be too high. There exist several topology control techniques such as localized geometrical structures [7]–[12], dynamic cluster techniques [13], [14] and power assignment protocols [15], [16]. In this paper, we focus on geometrical structures based methods.

Though many 2D geometrical structures have been proposed, surprisingly, there is not much study of 3D geometrical methods for topology control in 3D sensor networks, except for [17]–[22]. Bahramgiri *et al.* [17], Ghosh *et al.* [20] and Poduri *et al.* [21] proposed methods based on generalized CBTC algorithm [9] to preserve connectivity in 3D sensor networks, however, all of their 3D structures cannot bound the node degree, i.e., some nodes may have large unbounded number of neighbors. In addition, their construction methods are complex. Wang *et al.* [18], [19] and Kim *et al.* [22] then proposed a set of 3D structures based on Yao structures [7], [8] in 3D space. These 3D structures can be constructed locally, and they do have bounded node out-degree. However, none of them can bound the node in-degree. Possible unbounded in-degree at some nodes will often cause large overhead or contention at those nodes which may make them exhausted earlier than other nodes. Faced up with this challenge, in this paper we study how to efficiently construct 3D topology with bounded node degree to maintain connectivity, conserve energy and enable energy efficient routing.

The rest of this paper is organized as follows. Section II introduces the network model and desired properties of 3D topologies and Section III provides a review of related work. Section IV presents our algorithms to build 3D Yao-based topologies with bounded degree and the theoretical proofs of their nice properties. Section V shows the simulation results and Section VI concludes the paper.

II. Network Models and Assumptions

A 3D wireless sensor network consists of a set V of n wireless sensor nodes distributed in a 3D plane $\mathbb{R}^3$. Each node has the same *maximum transmission range* R. These nodes define a *unit ball graph* (UBG), or called *unit sphere graph*, in which there is an edge uv between two nodes u and v if and only if the Euclidean distance $\|uv\|$ between u and v in $\mathbb{R}^3$ is at most R. In other words, two nodes

The work of F. Li is partially supported by the National Natural Science Foundation of China (NSFC) under Grant 60903151. The work of Y. Wang is supported in part by the US National Science Foundation (NSF) under Grant No. CNS-0721666, CNS-0915331, and CNS-1050398, and the NSFC under Grant No. 61170212, and by Tsinghua National Laboratory for Information Science and Technology (TNList). Y. Wang (yu.wang@uncc.edu) is the corresponding author.

can always receive the signal from each other directly if the distance between them is no more than R. If there exists a link uv in UBG, v is a neighbor of u. All neighbors of u form its one-hop neighborhood, denoted as $N_{UBG}(u)$ or $N(u)$. The size of $N_{UBG}(u)$ is the node degree of u. We assume that each node knows its position information. By one-hop broadcasting, each node u can gather the location information of all nodes within its transmission range. As in the most common power-attenuation model, the power to support a link uv is assumed to be $\|uv\|^{\beta}$, where β is a real constant between 2 and 5 depending on the environment.

Topology control protocols aim to maintain a structure H from the original communication graph UBG that can preserve connectivity, optimize network throughput with power-efficient routing and conserve energy. The constructed topology H could be a directed or undirected graph. In the literature, the following desirable features of the structure are well-regarded and preferred in wireless sensor networks:

- *Connectivity:* To guarantee communications among all sensor nodes, the constructed topology H needs to be *connected*, i.e., there exists a path between any pair of nodes in the topology.
- *Bounded Node Degree:* It is also desirable that node degree (both in-degree and out-degree for directed graph) in H is small and upper-bounded by a constant. A small node degree reduces the MAC-level contention and interference, and may help to mitigate the well-known hidden and exposed terminal problems. In addition, a graph with a bounded node degree is always sparse, i.e., the total number of links is linear with the total number of nodes. A sparse graph conserves more energy in term of maintaining the constructed network topology.
- *Power Spanner:* A good network topology should be *energy efficient*, i.e., the total power consumption of the least energy cost path between any two nodes in final topology should not exceed a constant factor of the power consumption of the least energy cost path in original network [7]. Given a path $v_1 v_2 \cdots v_h$ connecting two nodes v_1 and v_h, the energy cost of this path is $\sum_{j=1}^{h-1} \|v_j v_{j+1}\|^{\beta}$. The path with the least energy cost is called the shortest path in a graph. A subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. The constant ρ is called the *power stretch factor*. A power spanner of the communication graph (e.g., UBG) is usually energy efficient for routing.
- *Localized Construction:* Due to limited resources at each wireless node, it is preferred that the topology can be constructed locally. Here, a topology can be constructed locally, if every node u can decide all edges incident on itself in the topology by only using the information of nodes within a constant hops of u.

III. Related Works

A. Topology Control in 2D Networks

With the objective of achieving energy efficiency and maintaining network connectivity, several localized geometrical structures have been proposed for topology control in 2D wireless networks, such as *local minimum spanning tree* (LMST) [10], *relative neighborhood graph* (RNG) [23], [24], *Gabriel graph* (GG) [23], *Yao graph* (YG) [7], [8], *cone-based topology control* (CBTC) [9], *Delaunay-based graph* [11], [12], and different combinations of these graphs [25]. By constructing such sparse topology structures, transmission powers of nodes can be minimized. As a result, the number of links in the constructed topology is significantly reduced comparing with that of the original communication graph which contains all links supported by the maximum transmission power. Among these 2D structures, some are planar structures (such as LMST, RNG, GG and Delaunay-based graphs), some are power spanners (such as GG, Yao graph, CBTC, and Delaunay-based graphs), and some are with bounded node degree (such as Yao graph).

Besides these localized geometrical structures, there are also other various techniques proposed by researchers for topology control in 2D sensor networks, such as how to construct a virtual backbone for routing [13], [14] and how to minimize the total transmission power while maintaining connectivity or other properties [15], [16].

B. Topology Control in 3D Networks

Although geometric topology control protocols have been well studied in 2D networks, the design of 3D topology control is surprisingly more difficult than the design in 2D. Current 2D methods cannot be directly applied in 3D networks. Wang *et al.* [18] proved that there is no embedding method mapping a 3D network into a 2D plane so that the relative scale of all edge length is preserved and all 2D geometric topology control protocols can still be applied for power efficiency. Thus, any simple mapping method from 3D to 2D does not work. On the other hand, many properties of 3D networks require additional computational complexity. Until recently, little research has been done on topology control for 3D wireless networks.

To solely achieve the connectivity of a 3D network, LMST [10] may be the best choice, since it is very sparse (with a bounded node degree) and can be easily constructed even for 3D networks and preserve connectivity. However, LMST may have very large power stretch factor. Similarly, RNG and GG can be easily extended to 3D, as shown in [18]. However, both of them do not have bounded node degree.

Bahramgiri *et al.* [17] generalized CBTC algorithm from 2D to 3D to preserve connectivity. Basically, each node u increases its transmission power until there is no empty 3D-cone with angle degree α, i.e., there exists at least a node in each 3D-cone of degree α centered at u, if $\alpha \leqslant \frac{2\pi}{3}$. This algorithm can be extended to ensure k-connectivity with $\alpha \leqslant \frac{2\pi}{3k}$. Even though this approach can guarantee connectivity, the gap detection algorithm applied to check the existence of the empty 3D-cone of degree α is very complicated. The time complexity of the gap detection algorithm at a node u is $O(d^3 \log d)$, where d is the node degree of u. Moreover, their method cannot bound node degree, as shown by [26].

Wang *et al.* [18] proposed a set of 3D Yao-based topologies (FiYG and FlYG), which can be constructed locally and efficiently. They proved some nice properties of these 3D Yao-based structures, e.g., bounded node out-degree and constant power stretch factor. Later, Wang *et al.* [19] also extended them to support fault tolerance.

Ghosh *et al.* [20] also presented two CBTC-based approaches for 3D wireless networks. Though the first approach, a heuristic based on 2D orthographic projections, can provide excellent performance in practice, it cannot guarantee connectivity for sure. In the second approach, a spherical Delaunay triangulation (SDT) is built to determine the existence of empty 3D cones. Although the second approach can guarantee connectivity of the network, the expense to construct the SDT is very high. Similarly, Poduri *et al.* [21] also used the spherical Delaunay triangulation to find the largest empty 3D cone in order to apply a CBTC-based topology control. The expense of SDT construction makes it inefficient in practice.

Recently, Kim *et al.* [22] proposed another localized Yao-based structure with Platonic solid (PYG). The basic idea is the same as the 3D Yao structures in [18] except for the partition method of 3D cones. To construct PYG, each node divides the 3D sphere neighborhood into k equal cones by using regular k-polyhedron and selects the neighbor that requires the lowest transmit power in each cone. The authors also consider the interference effects to adaptively choose the minimum transmit power and adjust the topology. However, such modification will break the connectivity and power spanner guarantees of 3D Yao structures.

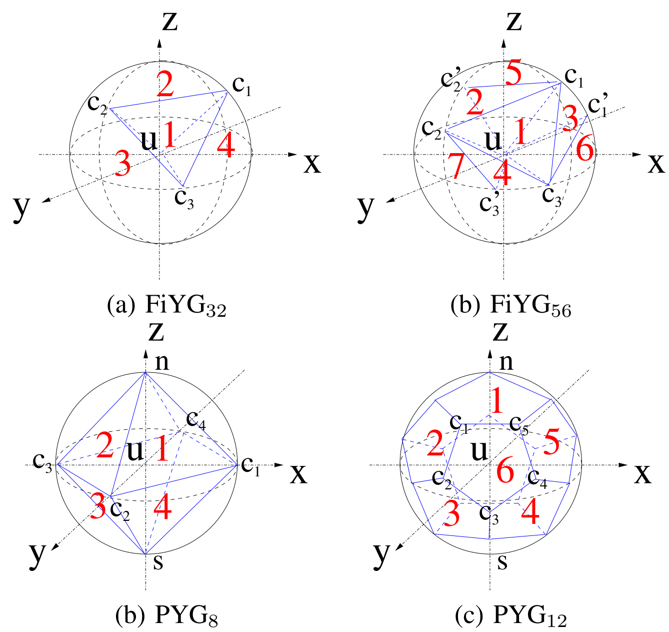
Overall, all of these existing 3D structures cannot achieve bounded node degree. Notice that even though 3D Yao structures (including FiYG, FlYG and PYG) can bound the node out-degree, their node in-degree could be as large as $O(n)$ where n is the number of nodes. Faced up with this challenge, in this paper we study how to efficiently construct 3D topologies with bounded node degree to maintain connectivity and conserve energy.

IV. Degree-bounded 3D Topologies: 3D Yao & Reverse Yao Graph and 3D Symmetric Yao Graph

In this section, we propose two general frameworks to build degree-bounded 3D topologies for wireless sensor networks. The proposed frameworks basically apply the existing 3D Yao structures via two techniques developed in [8], [11], [27] for 2D topology control protocols. In addition, we provide detail analysis on the degree bound and power stretch factor of the proposed new 3D topologies.

A. Review of Basic 3D Yao Structures

Our general frameworks to building degree-bounded 3D topologies are based on any existing 3D Yao structure (such as FiYG [18], FlYG [18] and PYG [22]). These 3D Yao structures use certain types of 3D cones to partition the transmission range (a sphere) of a node, and inside each 3D cone the node only keeps a link to the nearest neighbor. Since the number of such 3D cones is bounded by a constant k, all of them can bound the node out-degree by k. Here, k is a constant depending on which method and parameter are used. Basically, these structures can be categorized into two sets: fixed partition and flexible partition.

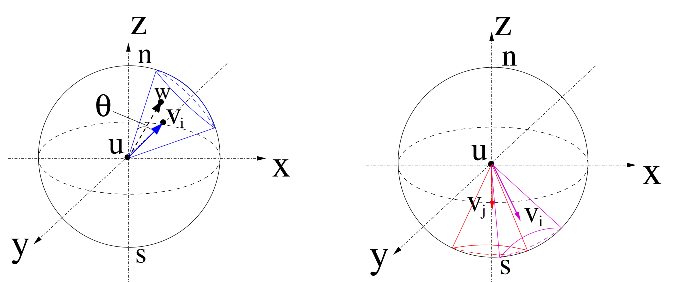


(a) FiYG_{32} (b) FiYG_{56} (b) PYG_8 (c) PYG_{12}

Figure 1. Definitions of 3D Yao Structures with fixed partitions: (a) and (b), partitions of $1/8$ of the ball in FiYG; (c) and (d), partitions using a octahedron or a dodecahedron for PYG.

3D Yao Structures based on Fixed Partition: In fixed partition, 3D cones from one node do not intersect with each other and the partition method is the same for all nodes. In [18], Wang *et al.* first proposed two methods to divides the transmission range of a node into certain number of 3D cones. Figure 1(a) and (b) illustrate these two methods, which divides the transmission ball into 32 and 56 cones respectively. For each cone, node u will choose the shortest edge $uv \in$ UBG among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph are denoted by FiYG_{32} and FiYG_{56} respectively. Notice that these cones in FiYG_{32} and FiYG_{56} are different and do not intersect with each other. In [22], Kim *et al.* then proposed another type of fixed partition method which divides the unit ball into k equal cones by using a regular k-polyhedron and selects the nearest neighbor in each cone. The resulting directed graph is denoted by PYG_k. Possible polyhedrons include tetrahedron, cube, octahedron, dodecahedron and icosahedron for $k = 4, 6, 8, 12, 20$ respectively. Figure 1(c) and (d) illustrate partition examples with a octahedron $k = 8$ and a dodecahedron $k = 12$. Notice that the cones in this method are with same shape/size and do not intersect with each other. All above methods based on fixed partition can be performed locally using 1-hop neighbor information and with $O(d)$ time, where d is the number of 1-hop neighbors.

3D Yao Structures based on Flexible Partition: In flexible partition, identical 3D cones with a top angle 2θ are used to partition the transmission ball and where to define these cones depends on the locations of neighbors around node u. In [18], Wang *et al.* proposed three different methods to perform the partition. However, they showed that the first method does not bound the node out-degree. Here, we just review their third method. Initially, all neighbors v_i of node u are *unprocessed* and ordered by the distance to u. The algorithm processes link uv from the shortest link and follows an ascending order. When it processes uv_i, it defines the 3D cone C_{uv_i} which uses uv_i as its axis (as shown in Figure 2(a)), adds the link uv_i, and marks all other links in C_{uv_i} as *processed*. We denote the final structure as FlYG$_\theta$ or FlYG when value of θ is clear. Algorithm 1 illustrates the detailed algorithm. The time complexity of this algorithm is $O(d \log d)$ due to the sorting. Notice that the 3D cones in this method are in the same size/shape and can intersect with each other (as in Figure 2(b)). By using a volume argument[1], Wang *et al.* showed that the node out-degree of FlYG is bounded by $k = \lceil \frac{2}{1-cos(\frac{\theta}{2})} \rceil$. If $\theta = \pi/4$, the degree bound is $\frac{2}{1-cos(\frac{\theta}{2})} \approx 26$, and if $\theta = \pi/6$, the degree bound is 58.



(a) 3D cone defined by uv_i (b) Possible overlapping of cones

Figure 2. Definitions of 3D Yao structures with flexible partitions.

B. General Frameworks to Build 3D Symmetric Yao Graph and 3D Yao & Reverse Yao Graph

Bounded out-degree from 3D Yao structures gives us advantages when apply several routing algorithms on these

[1]As shown in Figure 2(b), the angle between two neighbors in FlYG$_\theta$ must be larger than θ, i.e., $\angle v_i u v_j < \theta$.

Algorithm 1 Constructing 3D $FlYG$ for Node u

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.
Output: neighbors $N_{FlYG}(u)$ of u in the constructed FlYG.

Sort all neighbors $v_i \in N_{UBG}(u)$ by its length such that $\|uv_i\| \leq \|uv_{i+1}\|$, where $i = 1$ to $|N_{UBG}(u)|$.
Set PROCESSED(v_i) = 0 for all neighbor $v_i \in N_{UBG}(u)$.
for $i = 1$ to $|N_{UBG}(u)|$ **do**
  if PROCESSED$(v_i) = 0$ **then**
    As shown in Figure 2(a), let C_{uv_i} be the cone using uv_i as the axis and 2θ as the top angle.
    Keep v_i as a neighbor of u in FlYG, i.e., add v_i in $N_{FlYG}(u)$.
    Set PROCESSED$(w) = 1$ for every other neighbor w inside C_{uv_i}.
  end if
end for
return $N_{FlYG}(u)$

Algorithm 2 Building 3D Symmetric Yao Graph at Node u

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.
Output: neighbors $N_{SYG}(u)$ of u in the constructed SYG.

$N_{YG}(u)$ =3D-YAO-Structure($N_{UBG}(u)$).
Broadcast $N_{YG}(u)$ to all neighbors $N_{UBG}(u)$.
for all node $v \in N_{YG}(u)$ **do**
  if $u \in N_{YG}(v)$ **then**
    Keep v as a neighbor of u in SYG, i.e., add v in $N_{SYG}(u)$.
  end if
end for
return $N_{SYG}(u)$

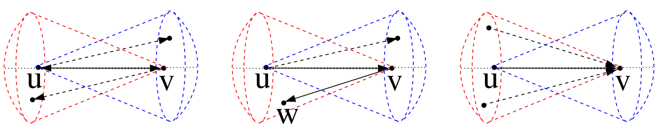
Algorithm 3 Building 3D Yao and Reverse Yao Graph at u

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.
Output: neighbors $N_{YYG}(u)$ of u in the constructed YYG.

$N_{YG}(u)$ =3D-YAO-Structure($N_{UBG}(u)$).
Broadcast $N_{YG}(u)$ to all neighbors $N_{UBG}(u)$.
Let $N^{in}_{YG}(u)$ be the set of u's incoming neighbors, i.e, all node v satisfying $u \in N_{YG}(v)$.
$N^{in}_{YYG}(u)$ =3D-YAO-Structure($N^{in}_{YG}(u)$).
Broadcast $N^{in}_{YYG}(u)$ to all neighbors $N_{UBG}(u)$.
for all node $v \in N_{YG}(u)$ **do**
  if $u \in N^{in}_{YYG}(v)$ **then**
    Keep v as a neighbor of u in YYG, i.e., add v in $N_{YYG}(u)$.
  end if
end for
return $N_{YYG}(u)$

structures. However, possible unbounded in-degree at some nodes will often cause large overhead or contention at those nodes which may make them exhausted earlier than other nodes. Therefore it is often imperative to construct a sparse network topology such that both the in-degree and the out-degree are bounded by a constant while it is still power spanner. Hereafter, we define a general function 3D-YAO-Structure() which can generate the neighbor set of 3D Yao structure at node u given the current neighbor set of u. We use YG to denote the generated 3D Yao structure.

The first set of 3D topologies is *3D Symmetric Yao Graph* (SYG), an undirected graph, which guarantees that the node degree is at most k. It first apply the 3D Yao structure to select the closest node in each 3D cone. An link uv is selected to graph SYG if and only if both u and v are selected to be kept by each other in YG, i.e., $v \in N_{YG}(u)$ and $u \in N_{YG}(v)$. See Figure 3(a) and (b) for illustrations. Algorithm 2 shows the framework. It is clear that only one-hop information is used and total $O(n)$ of messages are used. Thus, the SYG can be built locally and efficiently. Note that similar idea has been used in 2D networks [11], [27], [28].



(a) $uv \in$ SYG (b) $uv \notin$ SYG (c) $\overrightarrow{uv} \in$ YYG

Figure 3. Illustrations of 3D Yao Structures with bounded degree: (a) and (b) 3D Symmetric Yao Graph; (c) 3D Yao and Reverse Yao Graph.

The second set of 3D topologies is *3D Yao and Reverse Yao Graph* (YYG), a directed graph, which guarantees that both node in-degree and node out-degree are at most k. The basic idea is to apply reverse 3D Yao structure on YG to bound the node in-degree. Node v chooses a node u from each 3D cone, if there is any, so the incoming link $\overrightarrow{uv}$ in YG has the smallest length among all incoming links from YG in that cone as shown in Figure 3(c). Similar idea has been used for 2D networks by [6], [8], [29]. Algorithm 3 shows the detailed algorithm. 3D YYG can be built locally and efficiently with only 1-hop neighbor information and linear number of messages.

C. Performance Analysis of 3D SYG and 3D YYG

We are now ready for providing some analysis on the 3D structures built by our general frameworks. We will use two basic properties of the underlying 3D Yao structures: (1) the out-degree of 3D YG is bounded by k; and (2) if a link $uv \in$ UBG is not kept in 3D YG, there must exist a shorter link uw kept in 3D YG and $\angle vuw < \theta$. Here θ is the largest angle possible in a 3D cone in FiYG or the half of the top angle of the 3D cone in FlYG. For simplification, we assume the maximum transmission range $R = 1$.

Theorem 1: Both 3D SYG and 3D YYG are strongly connected if the original 3D UBG is connected and the angle parameter θ in 3D YG is bounded by $\pi/3$.

Proof: We first prove the connectivity of 3D SYG, which is equivalent to prove that there is a directed path from u to v in SYG for any two nodes u and v with $||uv|| \leq 1$. We prove this claim by an induction over the distance $||uv||$ between nodes u and v. First, note that the edge between the closest pair of nodes must be kept in SYG. Assume that the claim is true for all links less than $||uv||$. Now we consider nodes u and v. If uv is kept in SYG, the claim is true. If uv is not in SYG, there must a node w inside one of 3D cones at u or v who causes the deletion of uv. Assume w and v are in the same cone of u and $||uw|| < ||uv||$. Because the angle $\angle wuv$ is less than $\theta \leq \frac{\pi}{3}$, we have $||vw|| < ||uv||$. By induction there is a path from u to w and a path from w to v in SYG. Therefore a path from u to v exists in SYG. This finish the prove for SYG.

We next prove that SYG is a subgraph of YYG, which then can imply the connectivity of 3D YYG automatically. Assume that there exits a link uv in SYG but not in YYG. From the definition of SYG, we know link uv is selected by both u and v in 3D YG. Then if we apply reverse Yao structure on incoming neighbor of 3D YG (as Line 4 in Algorithm 3), uv will also be selected by node v. Thus, uv must be in YYG. This is a contradiction. Therefore, YYG is a supergraph of SYG and fully connected. ■

Theorem 2: The node degree of 3D SYG is bounded by k while both out-degree and in-degree of 3D YYG is bounded by k, where k is the degree bound of underlying 3D YG.

Proof: This is straightforward from the construction methods of SYG and YYG. Both methods first apply 3D YG. Since each node has at most k 3D cones during this construction, the out-degree is bounded by k. For 3D SYG, a link is kept only if both endpoint keeps it in 3D YG. Thus, the degree of SYG is obviously bounded by k. For 3D YYG, the second round of 3D YG is applied to incoming links, thus the in-degree is also bounded by k. ■

Theorem 3: The 3D SYG is not a power spanner of UBG, while 3D YYG is a power spanner of UBG when $\beta \geq 3$ and $\theta < \pi/3$.

Proof: The first half of this theorem can be directly obtained from a result by Grunewald, *et al.* [6]. They basically show how to construct a counter example of a 2D network in which SYG is not a power spanner. Since the 2D network is a special case of 3D networks, the same counter example works for 3D networks.

The proof of power spanner property of YYG is much challenging, even in 2D. Jia *et al.* [30] first proved that 2D YYG is a power spanner when $\theta \leq \pi/60$ (i.e., $k \geq 120$). It seems that their proof might be extended to 3D case, however, the node degree bound will be huge (larger than $\frac{4\pi/3}{2\pi(1-\cos(\theta/2))/3} \geq 5836$). Thus it is not very useful in practice. Schindelhauer *et al.* [31] then proved that 2D YYG is a power spanner with power spanning ratio $(8c+1)^2\frac{(2c)^\beta}{1-2^{(2-\beta)}}$ for $\beta > 2$ when $k > 6$. Here $c = \frac{1}{1-2\sin(\pi/k)}$. They proved this by first proving that 2D YYG is a weak c-spanner. In a weak c-spanner, between any two nodes there exists a path which remains within a disk or sphere of radius c-times the Euclidean distance between them. Their proof of weak spanner property of YYG can also be extended to 3D YYG with $\theta < \pi/3$. However, to further extend it to 3D power spanner, it requires $\beta \geq 3$. More specifically, 3D YYG is a power spanner with power spanning ratio $(8c+1)^3\frac{(2c)^\beta}{1-2^{(3-\beta)}}$ for $\beta > 3$ or $O(c^{12})$ for $\beta = 3$ when $\theta < \pi/3$. Therefore, we can claim that 3D YYG is a power spanner for $\beta \geq 3$ and $\theta < \pi/3$. When $2 \leq \beta < 3$, the power spanner property is still open. ■

V. Simulation

In order to evaluate the performance of our new 3D topologies with bounded degree, we conduct simulations in a simulator developed at our group by generating random 3D sensor networks. In our experiments, we randomly generate a set V of n wireless nodes and the UBG, then test the connectivity of UBG. If it is connected, we construct different localized 3D topologies proposed in this paper and some existing ones, and measure the node degree as well as power efficiency of these topologies. We repeat the experiment for multiple times and report the average or maximum values of these metrics in all of the simulations.

We evaluate the following localized 3D topologies:

- Sparse topologies: RNG and GG;
- 3D Yao structures: FiYG_{32}, PYG_8, PYG_{12}, $\text{FlYG}_{\pi/4}$, and $\text{FlYG}_{\pi/6}$;
- 3D Symmetric Yao structures: FiSYG_{32}, PSYG_8, PSYG_{12}, $\text{FlSYG}_{\pi/4}$, and $\text{FlSYG}_{\pi/6}$;
- 3D Yao & Reverse Yao structures: FiYYG_{32}, PYYG_8, PYYG_{12}, $\text{FlYTG}_{\pi/4}$, and $\text{FlYYG}_{\pi/6}$;

Here, *SYG and *YYG are the symmetric Yao structure and Yao & Reverse Yao structure based on the corresponding underlying 3D Yao structures, respectively.

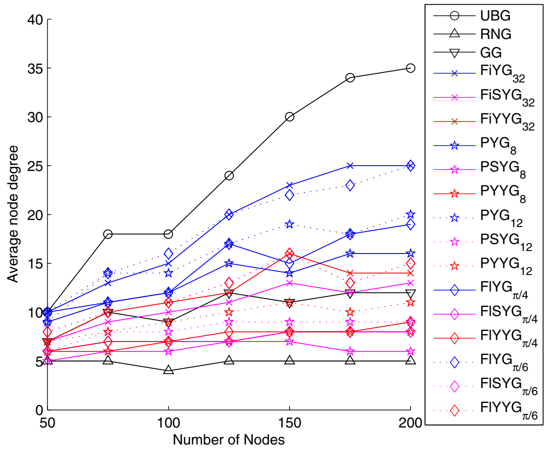
In the results presented here, we generate n random wireless nodes in a $10\times10\times10$ cube; the maximum transmission range R is set to $\sqrt{10}$ and the power constant $\beta = 2$, thus the maximum transmission power $P^{max} = R^2 = 10$. We vary the number of nodes n in the network from 50 to 200, where 100 vertex sets are generated for each case. The average and the maximum are computed over all these 100 vertex sets. All experimental results are plotted in Figure 4.

The node degree of wireless networks should not be too large to avoid interference, collision, and overhead. It should not be too small either: a low node degree usually implies that the network has a low fault tolerance and tends to increase the overall network power consumption as longer paths may have to be taken. Figure 4(a) shows all localized 3D topologies have lower average degrees compared with UBG and keep small degrees when the UBG becomes denser and denser as the number of nodes in the network increases. Clearly, Yao structures with larger number of 3D cones (larger k or smaller θ) lead to denser structures, while RNG is much sparser than most of Yao-based structures (but it is not a power spanner of UBG). Notice that the structures sparser than GG are Symmetric Yao structures and Yao & Reverse Yao structures of PYG_{12}, PYG_8 and $\text{FlYG}_{\pi/4}$ since these structures have smallest 3D cones. For the structures based on the same 3D Yao construction method, symmetric Yao structure is sparser than Yao & Reverse Yao structure. These can also be verified by Figure 4(b) (the maximum node degree). Figure 4(c) shows that the maximum node out-degree of these localized 3D-topologies are small. The out-degrees of all Yao structures are smaller than the theoretical bounds. Figure 4 (d) also gives the maximum node in-degree of these topologies which are a little bit larger than their out-degrees. It is clear that both symmetric Yao structures and Yao & Reverse Yao structures have smaller in-degrees than 3D Yao structures. Remember that theoretically they can bound the node in-degree. The results also showed that RNG and GG do not have large degrees and 3D Yao structures do no have large in-degree in this experiment, and the reason is that the nodes are distributed randomly in the area. In real life, the network may not be distributed randomly, so it is possible that RNG and GG have large degrees and 3D Yao structures have large in-degrees. Such examples and simulation results can be found in [8].

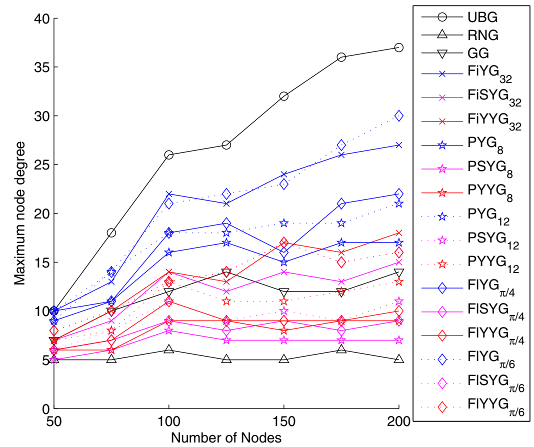
Besides connectivity, the most important design metric of wireless networks is perhaps power efficiency, because it directly affects both nodes and network lifetime. Figure 4(e) and 4(f) show all proposed structures have small power stretch factors even when the network is very dense. Notice that Yao structures based on PYG_8 and RNG have a little bit higher stretch factor than GG and other Yao-based structures, however, their maximum power stretch factors is still smaller than 5. The reason again is that the nodes are distributed randomly in the area. As we expected, GG has a power stretch factor of one and all power stretch factors of 3D Yao structures are smaller than their theoretical bounds if they have ones. Among all 3D Yao structures, $\text{FlYG}_{\pi/6}$ and $\text{FlYYG}_{\pi/6}$ have the smallest stretch factor, since they are the densest 3D Yao structures.

VI. Conclusion

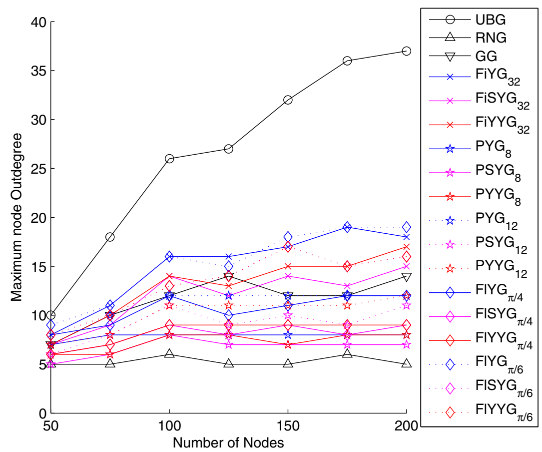
Topology control for 3D wireless sensor networks has been well studied recently and different 3D geometric topologies were proposed to achieve the sparseness and power efficiency. However, most of them cannot bound the node degree. Even though some of 3D structures based on Yao graph can bound the node out-degree, they may still lead to large in-degree at some nodes. Therefore, in this paper, we propose two general frameworks to build degree-bounded 3D topologies for wireless sensor networks. These frameworks can use all existing Yao-based 3D structures as



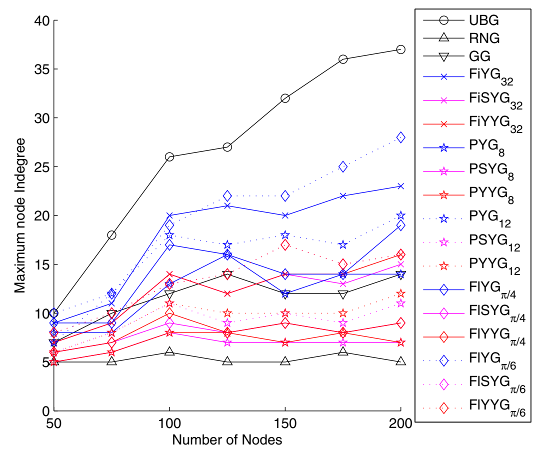
(a) avg-node-degree



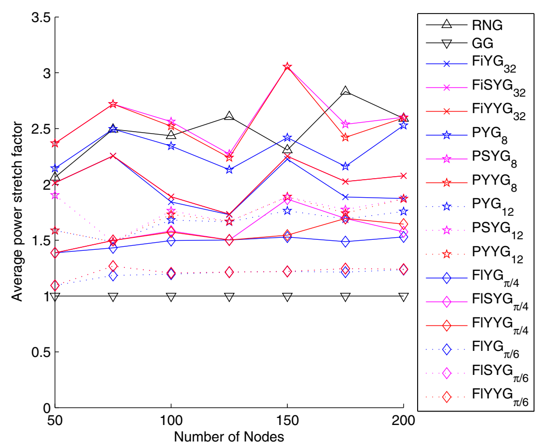
(b) max-node-degree



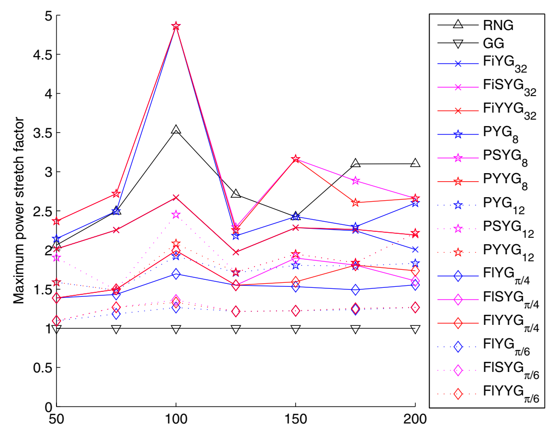
(c) max-node-out-degree



(d) max-node-in-degree



(e) avg-power-stretch-factor



(f) max-power-stretch-factor

Figure 4. Results when number of sensor nodes increasing from 50 to 200.

the underlying methods, and only use local information with linear number of messages. We show some of them can also guarantee the constant power stretch factor in 3D. Simulation results confirm good performance of new 3D topologies.

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