Energy-Efficient Restricted Greedy Routing for Three Dimensional Random Wireless Networks

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Abstract. In this paper, we investigate how to design energy-efficient localized routing in a large-scale three-dimensional (3D) wireless network. Several 3D localized routing protocols were proposed to seek either energy efficiency or delivery guarantee in 3D wireless networks. However, recent results [1, 2] showed that there is no deterministic localized routing algorithm that guarantees either delivery of packets or energy efficiency of its routes in 3D networks. In this paper, we focus on design of a simple localized routing method which can provide energy efficiency with high probability in a *randomly* deployed 3D network. In particular, we extend our previous routing method designed for 2D networks [3] to 3D networks. The proposed 3D routing method is a simple variation of 3D greedy routing and can guarantee energy efficiency of its paths with high probability in random 3D networks. We also study its asymptotic critical transmission radius to ensure the packet delivery with high probability in random 3D networks. Simulation results confirm our theoretical results.

1 Introduction

Recently, three-dimensional (3D) wireless network has received significant attention [4–8], due to its wide range of potential applications (such as underwater sensor networks [9]). However, the design of networking protocols for 3D wireless networks is surprisingly more difficult than that for 2D networks. In this paper, we focus on one particular problem in 3D networks: energy efficient localized geographic routing.

In localized geographic routing, the forwarding decision is made by the intermediate node based solely on its local information. Without the route discovery phase and maintenance of routing tables, it enjoys the advantages of lower overhead and higher scalability than other traditional routing protocols. This makes it very suitable for 3D networks. *Greedy routing* is one of the most popular localized routing methods, in which a packet is greedily forwarded to the closest node to the destination in order to minimize the average hop count. Greedy routing can be easily extended to 3D case. Actually, several under-water routing protocols [6, 10] are just variations of 3D greedy routing.

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However, to guarantee *packet delivery* or *energy efficiency* of 3D routing is not straightforward and very challenging. Simple greedy routing may fail to reach the destination when falls into a local minimum (a node without any "better" neighbors). Face routing [11] can be used on planar topology to recovery from local minimum and guarantee the delivery in 2D networks, but there is no planar topology concept any more in 3D networks. In fact, Durocher *et al.* [1] recently proved that there is *no* deterministic localized routing algorithm for 3D networks that guarantees the delivery of packets. On the other hand, even a localized routing method can find the route to deliver the packet, it may not guarantee the energy efficiency of the path, *i.e.*, the total power consumed compared with the optimal could be very large in the worst case. Several energy-aware localized 2D routing protocols [12–14] already took the energy concern into consideration, but none of them can theoretically guarantee the energy-efficiency of their routes. This is true for all existing 3D localized routing methods too. Recently, Flury and Wattenhofer [2] showed an example of a 3D network (Figure 1 of [2]) where the path found by *any* localized routing protocol to connect two nodes s and t has energy consumption asymptotically at least $\Theta(d^3)$ in the worst case. Here d is the optimal energy consumption to connect s and t.

Therefore, in this paper, we focus on the design of a simple localized routing method which can provide energy efficiency with high probability in a *randomly* deployed 3D network. In particular, we extend our previous routing method designed for 2D networks [3] to 3D networks. The proposed 3D routing method is a simple variation of 3D greedy routing and can guarantee energy efficiency of its path with high probability if it finds one in random 3D networks. To ensure that our routing method can find paths with high probability, we also study its asymptotic *critical transmission radius* (CTR) in random 3D networks. We prove that for a 3D network formed by nodes, that are generated by a Poisson point process of density n over a convex compact region of unit volume, the CTR for our proposed 3D routing is *asymptotic almost sure* (a.a.s.) at most $\sqrt[3]{\frac{3\beta \ln n}{4\pi n}}$ for any $\beta > \beta_0$ and at least $\sqrt[3]{\frac{3\beta \ln n}{4\pi n}}$ for any $\beta < \beta_0$. Here, $\beta_0 = \frac{2}{1-cos\alpha}$ where α is an parameter used by our routing method.

The rest of the paper is organized as follows. Section 2 presents our network model and a formal definition of CTR. Section 3 gives our proposed 3D localized routing protocol and derived bounds on its CTR. Section 4 shows our simulation results and Section 5 summarizes the paper.

2 Preliminaries

Network Model and Assumptions: We consider a set V of n wireless devices (called nodes hereafter) uniformly distributed in a compact and convex 3D region $\mathbb{D}$ with unit-volume in $\mathbb{R}^3$. By proper scaling, we assume the nodes are represented by a Poisson point process $\mathcal{P}_n$ of density n over a unit-volume cube $\mathbb{D}$. Each node knows its position information and has a uniform transmission radius r (or r_n). Then the communication network is modeled by a unit disk graph $G(V, r)$, where two nodes u and v are connected if and only if their Euclidean distance is at most r. Hereafter, we use $\|u - v\|$ to denote the Euclidean distance between u and v. For a link $uv \in G(V, r)$, we use $\|uv\|$

to denote its length. We further assume that the energy needed to support the transmission of a unit amount of data over a link uv is $\mathbf{e}(\|uv\|)$, where $\mathbf{e}(x)$ is a non-decreasing function on x. $|A|$ is shorthand for the volume of a measurable set $A \subset \mathbb{R}^3$. An event is said to be *asymptotic almost sure* if it occurs with a probability converges to one as $n \to \infty$. To avoid trivialities, we assume n to be sufficiently large if necessary.

Critical Transmission Radius for Greedy-based Routing: In any greedy-based routing, the packet may be dropped by an intermediate node when it could not find any of its neighbors that is "better" than itself. One way to ensure that the routing is successful for every source-destination pair is during the topology control phase each node is set with a sufficiently large transmission radius such that every intermediate node will always find a better neighbor. Critical transmission radius for routing algorithm is first studied by [15]. A routing method $\mathcal{A}$ is *successful* over a network G if the routing method $\mathcal{A}$ can find a path for any pair of source and destination nodes. Then we can define the critical transmission radius (CTR) of $\mathcal{A}$ as follows:

Definition 1. *Given a routing method $\mathcal{A}$ and a set of wireless nodes V, the critical transmission radius, denoted as $\rho_{\mathcal{A}}(V)$, for successful routing of $\mathcal{A}$ over V is the minimum transmission radius r such that method $\mathcal{A}$ over the network $G(V, r)$ is successful.*

The subscript $\mathcal{A}$ will be omitted from $\rho_{\mathcal{A}}(V)$ if it is clear from the context. In [15], Wan *et al.* derived the CTR of 2D greedy routing in random networks. Recently, Wang *et al.* [8] also derived the CTR of 3D greedy routing in random 3D networks. In this paper, we will use similar techniques from [8, 15] to study the CTR of our 3D routing.

3 Energy-Efficient Restricted 3D Greedy Routing

In this section, we present details of our energy-efficient localized 3D routing method and theoretical analysis on its critical transmission radius in 3D random networks.

3.1 Routing Method

Our energy-efficient localized 3D routing method is a variation of classical 3D greedy routing and an extension of a localized routing method [3] we designed for 2D networks. In 3D greedy routing, current node u selects its next hop neighbor based purely on its distance to the destination, i.e., it sends the packet to its neighbor who is closest to the destination. However, such choice might not be the most energy-efficient link locally, and the overall route might not be globally energy-efficient too. Therefore, our routing method use two concepts *energy mileage* and *restricted region* to refine the choices of forwarding nodes in 3D greedy routing.

Energy Mileage: Given a energy model $\mathbf{e}(x)$, *energy mileage* is the ratio between the transmission distance and the energy consumption of such transmission, *i.e.*, $\frac{x}{\mathbf{e}(x)}$. Let $\mathbf{r}_0$ be the value such that $\frac{\mathbf{r}_0}{\mathbf{e}(\mathbf{r}_0)} = \max_x \frac{x}{\mathbf{e}(x)}$. We call $\mathbf{r}_0$ as the *maximum energy mileage distance*[3] under energy model $\mathbf{e}(x)$. We assume that the energy mileage $\frac{x}{\mathbf{e}(x)}$ is an increasing function when $x < \mathbf{r}_0$ and a decreasing function when $x > \mathbf{r}_0$.

[3] Here, we assume that $d(\frac{\mathbf{e}(x)}{x})/dx$ is monotone increasing, thus, $\mathbf{r}_0$ is unique.

This assumption is true for most of commonly used energy models. For example, if $\mathbf{e}(\|uv\|) = \|uv\|^2 + c$ is the energy used by sending message from u to v, the maximum energy mileage distance $\mathbf{r}_0 = \sqrt{c}$. Our 3D localized routing greedily selects the neighbor who can maximize the energy mileage as the forwarding node.

Restricted Region: Instead of selecting the forwarding node from all neighbors of current node u (a unit ball in 3D as shown in Figure 1(a)), our 3D routing method prefers the forwarding node v inside a smaller restricted region. The region is defined inside a 3D cone with an angle parameter $\alpha < \pi/3$, such that angle $\angle vut \leq \alpha$, as shown in Figure 1(b). The use of α (restricting the forwarding direction) is to bound the total distance of the routing path. Then the restricted region is a region inside this 3D cone and near the maximum energy mileage distance $\mathbf{r}_0$, such that every node v inside this area satisfies $\eta_1 \mathbf{r}_0 \leq \|uv\| \leq \eta_2 \mathbf{r}_0$, as shown in Figure 1(b). Here, η_1 and η_2 are two constant parameters. This can help us to prove the energy-efficiency of the route.

Notice that both these ideas are not completely new. Restricted region with an angle has been used in some localized routing methods, such as nearest/farthest neighbor routing [16], while concepts similar to energy mileage have been used in some energy-aware localized routing methods [12,14]. However, combining both of these techniques to guarantee energy efficiency is first done in our previous work [3] for 2D networks. In this paper, we further adapt them into 3D routing.

Our energy-efficient localized 3D routing protocol is given in Algorithm 1. There are four parameters used by our method. Three adjustable parameters $0 < \alpha < \frac{\pi}{3}$ and $\eta_1 < 1 < \eta_2$ define the restricted region, while $\mathbf{r}_0$ is the best energy mileage distance based on the energy model. For example, the following setting of these parameters can be used for energy model $\mathbf{e}(x) = x^2 + c$: $\alpha = \frac{\pi}{4}$, $\mathbf{r}_0 = \sqrt{c}$, $\eta_1 = 1/2$ and $\eta_2 = 2$. Hereafter, we denote the routing algorithm, *energy-efficient restricted greedy*, as ERGrd if *no* greedy routing (Grd) is used when no node v satisfies that $\angle vut \leq \alpha$. If Grd is applied afterward, then the routing protocol is denoted ERGrd+Grd. Notice that if Grd fails to find a forwarding node, randomized scheme [2] could also be applied.

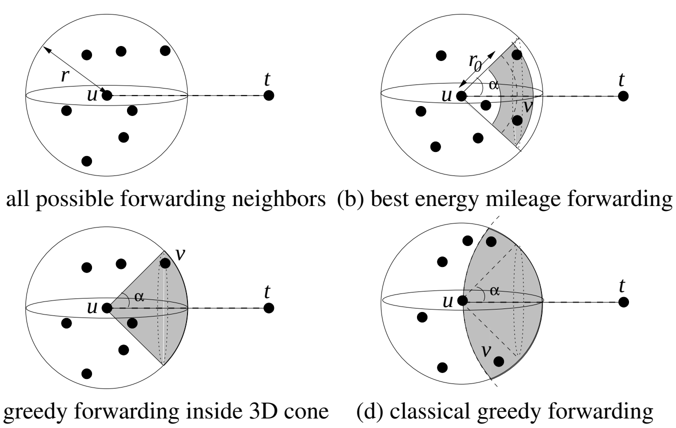


Fig. 1. Illustrations of our 3D routing: (a) energy-efficient forwarding in the restricted region, (b) greedy forwarding in the 3D cone, (c) greedy forwarding when the 3D cone is empty.

Algorithm 1 Energy-Efficient Restricted 3D Greedy Routing

while node u receives a packet with destination t **do**
  if t is a neighbor of u **then**
    Node u forwards the packet to t directly.
  else if there are neighbors inside the restricted region and $\mathbf{r}_0 < r$ **then**
    Node u forwards the packet to the neighbor v such that its energy mileage $\frac{\|uv\|}{\mathbf{e}(\|uv\|)}$ is maximum among all neighbors w inside the restricted region, as shown in Figure 1(b).
  else if there are neighbors inside the 3D cone **then**
    Node u finds the node v inside the 3D cone (Figure 1(c)) with the minimum $\|t - v\|$.
  else
    Greedy routing (Figure 1(d)) is applied, or the packet is simply dropped.
  end if
end while

The path efficiency of ERGrd is given by the following two theorems. The detail proofs of these two theorems are exactly the same with the proofs of Theorems 1-3 in [3] for 2D network, thus are ignored here.

Theorem 1. *When ERGrd routing indeed finds a path $\boldsymbol{P}_{ERGrd}(s,t)$ from the source s to the target t, the total Euclidean length of the found path is at most $\delta\|t - s\|$ where $\delta = \frac{1}{1-2\sin\frac{\alpha}{2}}$, thus, a constant factor of the optimum.*

Theorem 2. *When ERGrd routing indeed finds a path $\boldsymbol{P}_{ERGrd}(s,t)$ from the source s to the target t, the total energy consumption of the found path is within a constant factor σ of the optimum. When $\mathbf{r}_0 \geq r$, σ depends on α; otherwise, depends on η_1, η_2 and α.*

3.2 Critical Transmission Radius of 3D ERGrd Routing

Notice that ERGrd routing may fail, as all other greedy-based methods do, when an intermediate node cannot find a better neighbor to forward the packet. We now study the critical transmission radius for ERGrd routing in random wireless networks. Given a set of nodes V distributed in a region $\mathbb{D}$, the critical transmission radius $\rho(V)$ for successful routing by 3D ERGrd is

$$\max_{u,v} \min_{w:\,\angle wuv \leq \alpha} \|w - u\|. \tag{1}$$

By setting the $r = \rho(V)$, ERGrd can always find a forwarding node inside the 3D cone region, thus can guarantee its packet delivery. In this section, we prove a similar result as in [3, 8, 15] for our restricted 3D greedy routing method, 3D ERGrd.

Theorem 3. *Let $\beta_0 = \frac{2}{1-cos\alpha}$ and $n\left(\frac{4}{3}\pi r_n^3\right) = \beta \ln n$ for some $\beta > 0$. Then,*

1. *If $\beta > \beta_0$, then $\rho\left(\mathcal{P}_n\right) \leq r_n$ is a.a.s..*
2. *If $\beta < \beta_0$, then $\rho\left(\mathcal{P}_n\right) > r_n$ is a.a.s..*

Here, $\beta_0 = \frac{4\pi/3}{2\pi(1-cos\alpha)/3} = \frac{2}{1-cos\alpha}$ is the ratio between the volume of a unit ball and the volume of a 3D cone (the forwarding region) inside the ball. Next, we present the detailed proofs for two parts of this theorem. To simplify the argument, we ignore boundary effects by assuming that there are nodes outside $\mathbb{D}$ with the same distribution. So, if necessary, packets can be routed through those nodes outside $\mathbb{D}$.

Upper Bound of Theorem 3: To prove the upper bound, it is sufficient to show that when each node has a transmission radius r_n satisfying the above condition and $\beta > \beta_0$, for every pair of nodes u and v there is always a node $w \in \mathcal{P}_n$ such that $\angle wuv \leq \alpha$ and $\|w-u\| \leq r_n$, *i.e.*, any intermediate node u can find a "better" neighbor w towards the destination node v. Given a point distribution $\mathcal{P}_n$, let $\mathcal{S}(\mathcal{P}_n, r_n)$ be the minimum number of such neighboring nodes w that can be chosen by any intermediate node u for any possible destination v. As proved in [15], it suffices to prove that the cardinality $|\mathcal{S}(\mathcal{P}_n, r)| > 0$. Instead, we now prove a stronger result shown in the following lemma.

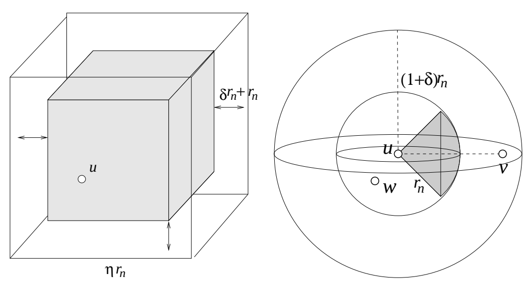
Lemma 1. *Suppose that* $n\left(\frac{4}{3}\pi r_n^3\right) = \beta \ln n$ *for some* $\beta > \beta_0$*. Then for any constant* $\beta_1 \in (\beta_0, \beta)$*, it is a.a.s. that* $S(\mathcal{P}_n, r_n) > \mathcal{L}\left(\frac{\beta_1}{\beta_0}\right)\ln n$*. Here* $\mathcal{L}(x) = x\phi^{-1}(1/x)$ *for* $x > 0$ *and* $\phi(x) = 1 + x\ln x - x$ *for* $x > 0$*.*

PROOF. Given a node u, the region that node u can choose its neighbor to forward data is a 3D cone with angle 2α, as shown in Figure 1(c). Let $\mathbf{Y}$ denote this 3D cone and d denote its diameter (*i.e.*, the largest distance between any two points inside it). Clearly $d = r_n$ when $\alpha \leq \frac{\pi}{6}$, and $d = 2\sin\alpha \cdot r_n$ when $\frac{\pi}{6} \leq \alpha < \frac{\pi}{3}$. Thus, $d < \sqrt{3}r_n$.

Assume that the space is partitioned into 3D grids (equal-size cubes) of side length η, which we call it η-tessellation of space. Here, we consider an εd-tessellation, where $\varepsilon = \frac{4}{27\beta_0}(1 - \frac{\beta_2}{\beta})$. A *polycube* is defined as the set of cubes that intersect with a convex and compact region, *e.g.*, $\mathbf{Y}$. Notice that when the grid-partition shifts, we will have different polycubes for the fixed region $\mathbf{Y}$. A polycube in a η-tessellation is said to have a *span* s if it can be contained in a cube of side-length $s \cdot \eta$. We are only interested in polycube that has span at most $\frac{1}{\varepsilon}$ and volume at least a certain fraction of $\frac{4\pi}{3}r_n^3$. Assume that, given $\mathbf{Y}$, there are I_n different such polycubes that are completely contained inside, with span at most $\frac{1}{\varepsilon}$ and volume at least $\frac{\beta_2}{\beta_0\beta}\frac{4\pi}{3}r_n^3 = (\frac{\beta_2}{\beta_0})\frac{\ln n}{n}$. For ith such polycubes, let X_i denote the number of nodes of V contained inside. Then X_i is a Poisson RV with rate at least $\left(\frac{\beta_2}{\beta_0}\right)\ln n$. Since the number of cubes in $\mathbb{D}$ is $O\left(\left(\frac{1}{\varepsilon d}\right)^3\right) = O\left(\frac{n}{\ln n}\right)$, by Lemma 3 of [8], $I_n = O\left(\frac{n}{\ln n}\right)$. By Lemma 6 of [15], it is a.a.s. that $\frac{\min_{i=1}^{I_n} X_i}{\ln n} \geq \mathcal{L}\left(\frac{\beta_2}{\beta_0}\right) > \mathcal{L}\left(\frac{\beta_1}{\beta_0}\right)$.

To prove the lemma, it is sufficient to show that $\mathcal{S}(\mathcal{P}_n, r) \geq \min_{i=1}^{I_n} X_i$. In other words, we only need to show that for any $\mathbf{Y}$, it contains a polycube P has span at most $\frac{1}{\varepsilon}$ and volume at least $\frac{\beta_2}{\beta_0\beta}\frac{4\pi}{3}r_n^3$. For a 3D cone $\mathbf{Y}$, let P denote the polycube induced by $\mathbf{Y}_{-\sqrt{3}\varepsilon d}$. Here $\mathbf{Y}_{-x}$ denotes the region of $\mathbf{Y}$ whose points are of distance at least x from the boundary of $\mathbf{Y}$. Then, $P \subseteq \mathbf{Y}$, and the span of P is at most $\left\lceil\frac{d-2\sqrt{3}\varepsilon d}{\varepsilon d}\right\rceil + 1 < \frac{1}{\varepsilon}$. By Lemma 2 in [8] and the fact that $|\mathbf{Y}| = \frac{4}{3}\pi r_n^3\frac{1}{\beta_0} > \frac{4}{9\sqrt{3}}\pi d^3\frac{1}{\beta_0}$, we have $|P| \geq \left|\mathbf{Y}_{-\sqrt{3}\varepsilon d}\right| \geq |\mathbf{Y}| - \pi d^2\left(\sqrt{3}\varepsilon d\right) = |\mathbf{Y}| - \sqrt{3}\varepsilon\pi d^3 > |\mathbf{Y}| - \frac{27\beta_0}{4}\varepsilon|\mathbf{Y}| = |\mathbf{Y}|\left(1 - \frac{27\beta_0}{4}\varepsilon\right) = \frac{\beta_2}{\beta}|\mathbf{Y}| = \frac{\beta_2}{\beta_0}\left(\frac{4}{3}\pi r_n^3\right)\frac{1}{\beta}$. This completes the proof of the lemma. □

Lower Bound of Theorem 3: We now show that, if $r_n = \sqrt[3]{\frac{3\beta\ln n}{4\pi n}}$ for any $\beta < \beta_0$, *a.a.s.*, there are two nodes u and v such that we cannot find a node w for forwarding by node u, *i.e.*, there does not exist node w inside the 3D cone. Again we partition the space using equal-size cubes (called cells) with side-length ηr_n for a constant $0 < \eta$ to be specified later. Thus the number of cells, denoted by I_n here, that are fully contained inside the compact and convex region $\mathbb{D}$ with unit volume, is $\Theta(\frac{1}{\eta^3 r_n^3}) = \Theta(\frac{n}{\ln n})$. Let $E_{u,v}$ denote the event that no forwarding node w (in the 3D cone) exists for node u to reach node v. Then to prove our claim, it is equivalent to prove that the probability of at least one of the event $E_{u,v}$ happens *a.a.s.*, *i.e.*, $1 - \mathbf{Pr}(\text{none of event } E_{u,v} \text{ happens})$. Since the events $E_{u,v}$ are not independent for all pairs u and v, we will only consider a special subset of events that are independent. Consider any cell produced by the 3D grid partition that are contained inside $\mathbb{D}$. For each cell, we draw a shaded cube with side-length $(\eta - 2(1+\delta))r_n$ and it is of distance $(1+\delta)r$ to the boundary of the cell, as shown in Figure 2 (a). We only consider the case when node u is located in this shaded cube. We also restrict the node v to satisfy that $r_n < \|u - v\| \leq (1+\delta)r_n$, *i.e.*, in the torus region in Figure 2 (b). Clearly, node v will also be inside this cell, and the shaded 3D cone where the possible forwarding node could locate is also inside this cell. Thus, events E_{u_1,v_1} and E_{u_2,v_2} are independent if u_1 and u_2 are selected as above from different cells.



(a) selection of u (b) no forwarding node

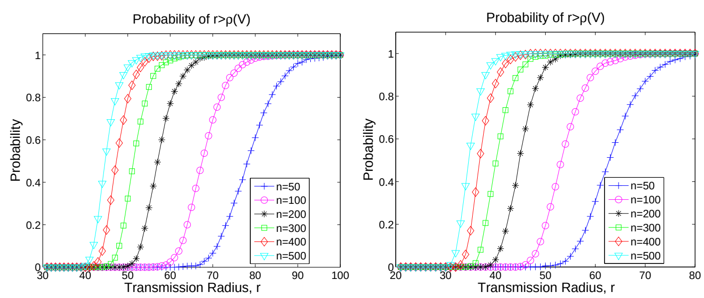
Fig. 2. Illustrations of the proof of lower bound: (a) a cubic cell and the region where we select a node u; (b) the event that node u cannot find a forwarding node w to reach a node v.

For each cell i, we compute the probability that event E_{u_i,v_i} happens, where u_i is selected from the shaded cube of cell i and v_i is selected such that $r_n < \|v_i - u_i\| \leq (1+\delta)r_n$. Recall that for any region A, the probability that it is empty of any nodes is $e^{-n|A|}$. Clearly, the probability that node u_i exists is $1 - e^{-n(\eta-2-2\delta)^3 r_n^3}$ since the shared cube has volume $(\eta-2-2\delta)^3 r_n^3$; the probability that node v_i exists is $1 - e^{-n\frac{4}{3}\pi((1+\delta)^3-1)r_n^3}$ since the torus has volume $\frac{4}{3}\pi((1+\delta)^3-1)r_n^3$. Given node u_i and v_i, the probability that event E_{u_i,v_i} happens is $e^{-n\frac{2}{3}\pi(1-cos\alpha)r_n^3} = e^{-\beta/\beta_0 \ln n} = n^{-\beta/\beta_0}$. Consequently, event $E_{u,v}$ happens for some node pairs u_i and v_i is $\mathbf{Pr}(E_{u_i,v_i}) \geq (1 - e^{-n(\eta-2-2\delta)^3 r_n^3})(1 - e^{-n\frac{4}{3}\pi((1+\delta)^3-1)r_n^3})n^{-\beta/\beta_0} = (1 - n^{-\beta(\eta-2-2\delta)^3 3/4\pi})(1 - n^{-\beta((1+\delta)^3-1)})n^{-\beta/\beta_0}$. Thus, the probability that ERGrd routing fails to find a path for some source/destination pairs is $\mathbf{Pr}(\text{at least one of events } E_{u,v} \text{ happens}) \geq \mathbf{Pr}(\text{at least one of } E_{u_i,v_i} \text{ happens}) = 1 - \mathbf{Pr}(\text{none of } E_{u_i,v_i} \text{ happens}) = 1 - (1 - \mathbf{Pr}(E_{u_i,v_i}))^{I_n} = 1 - e^{I_n \cdot \ln(1 - \mathbf{Pr}(E_{u_i,v_i}))} \geq 1 - e^{-I_n \cdot \mathbf{Pr}(E_{u_i,v_i})}$. Notice that $I_n \cdot \mathbf{Pr}(E_{u_i,v_i}) = \Theta(\frac{n}{\ln n})(1 - n^{-\beta(\eta-2-2\delta)^3 3/4\pi})(1 - n^{-\beta((1+\delta)^3-1)})n^{-\beta/\beta_0} \simeq \frac{n^{1-\beta/\beta_0}}{\ln n}$, which goes to ∞ as $n \to \infty$ when $\beta < \beta_0$, $\eta - 2 - 2\delta > 0$, and $\delta > 0$. This can be easily satisfied, *e.g.*, $\delta = 1$, $\eta = 5$. Thus, $\lim_{n\to\infty} 1 - e^{-I_n \cdot \mathbf{Pr}(E_{u_i,v_i})} = 1$. This completes the proof.

4 Simulation

In this section, we study performance of our ERGrd routing in random 3D networks via extensive simulation.

Critical Transmission Radius of ERGrd Routing: To confirm our theoretical analysis, we conduct simulations to study the real values of CTR of ERGrd routing method in random 3D networks. We randomly generate 1000 networks with n nodes in a $100 \times 100 \times 100$ cubic region, where n is from 50 to 500. For each network V, we compute the CTR $\rho(V)$ of ERGrd by using Equation (1). Figure 3 show the probability distribution function of $\rho(V)$ when $\alpha = \pi/6$ and $\alpha = \pi/4$. It is clear that the CTR satisfies a transition phenomena, *i.e.*, there is a radius r_0 such that ERGrd can successfully deliver all packets when $r > r_0$ and can not deliver some packets when $r < r_0$. Notice that the transition becomes faster when the number of nodes increases. This confirms our theoretical analysis on the existence of CTR. In addition, from the figures, we can find that larger node density always leads to smaller value of CTR. The practical value of $\rho(V)$ is larger than the theoretical bound in our analysis, since the theoretical bound is standing for $n \to \infty$. Compared the two cases with $\alpha = \pi/6$ and $\pi/4$, larger CTR is required if smaller restricted region (*i.e.* smaller α) is applied.



(a) PDF of ERGrd-30 (b) PDF of ERGrd-45

Fig. 3. PDF curves of 3D ERGrd routing when $\alpha = \pi/6$ ($30°$) and $\alpha = \pi/4$ ($45°$).

Network Performance of ERGrd Routing: We implement the classic 3D greedy routing (Grd) and variations of our proposed restricted greedy routing (specifically, RGrd-30, ERGrd-45, ERGrd+Grd-30, and ERGrd+Grd-45, where 30 or 45 is the degree value of α.) in our simulator. We assume that the energy consumption of a link uv is $\mathbf{e}(\|uv\|) = \|uv\|^2 + c$, where $c = r^2/4$. The values of η_1 and η_2 are $1/2$ and 2. By setting various transmission radii, we generate 100 connected random networks with 100 wireless nodes again in a $100 \times 100 \times 100$ cubic region. We randomly select 100 source-destination pairs for each network and test five greedy-based 3D routing. All results presented hereafter are average values over all routes and networks.

Figure 4 illustrates the average delivery ratios of the five routing methods. Clearly, the delivery ratio increases when r increases. After r is larger than a certain value, it always guarantees the delivery. Notice that ERGrd methods without Grd backup have lower delivery ratio under the same circumstance, since they have smaller region to select the next hop node. With Grd backup, the delivery ratios of ERGrd+Grd methods are almost the same with those of Grd (simple 3D greedy).

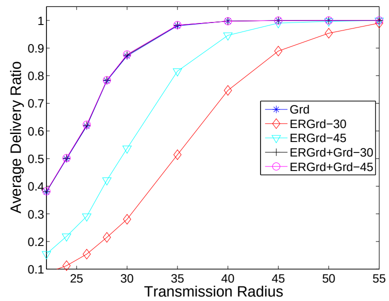
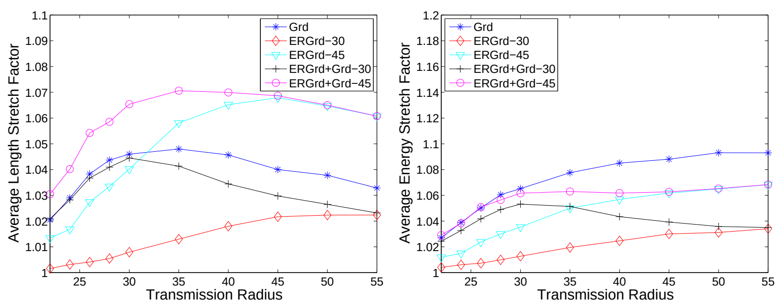


Fig. 4. Average delivery ratios of 3D ERGrd routing methods in random networks.



(a) Length Stretch Factor (b) Energy Stretch Factor

Fig. 5. Path efficiency (length stretch factor and energy stretch factor) of 3D ERGrd routing methods in random networks.

Figure 5(a) and (b) illustrate the average length stretch factors and energy stretch factors of all routing methods, respectively. Here, the *length/energy stretch factor* of a path from node s to node t is the ratio between the total length/energy of this path and the total length/energy of the optimal path connecting s and t. Smaller stretch factor of a routing method shows better path efficiency. For the length stretch factor, the ERGrd with $\alpha = \pi/6$ has the best length efficiency. It is surprising that with $\alpha = \pi/4$ the length of ERGrd path could be longer than simple greedy. However, when considering the energy efficiency, all ERGrd methods can achieve better path efficiency than simple greedy method. Notice that smaller restricted region leads to better path efficiency, however it also has lower delivery ratio. Therefore, it is a trade-off between path efficiency and packet delivery. It is also clear that when the network is dense (with large transmission radius), ERGrd and ERGrd+Grd are almost the same, since ERGrd can always find nodes inside the 3D cone.

5 Conclusion

In this paper, we propose a simple localized routing protocol ERGrd for 3D networks. The proposed method achieves the energy efficiency by limiting its choice inside a restricted region and picking the node with best energy mileage. We then theoretically derive the critical transmission radius of our proposed 3D routing for random 3D networks. This provides the insight about how large the transmission radius should be set for our method to guarantee the delivery of packets between any two nodes. We also conduct extensive simulations to confirm our theoretical results.

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