**Exercise 6.1 Insertion and Deletion in B*-trees**

Please insert the following keys in the given order into an empty B*-tree of class \( \tau(2, 2, h^*) \):

2, 6, 17, 20, 24, 25, 27, 29, 30, 31, 5, 21, 40, 45, 50, 70

a) Draw the tree before the first and all consecutive split operation

b) Delete the following keys 25, 6, 5, and 20. Draw the tree after each deletion.

**Solution:**

Insert: 2, 6, 17, 20

Insert: 24, 25, 27

Insert: 29, 30, 31

Insert: 32, 5, 21

Insert: 1, 40, 45

Delete: 25

Delete: 6, 5

Delete: 20

**Exercise 6.2 Height of B-Tree and B*-Tree: Upper and Lower Bounds**

For the B-tree and the B*-tree, please derive analytic formulae which enable the determination of the upper and lower bound for the height \( h \) of the tree when \( k \), \( k^* \), and \( n \) (number of elements in the tree) are given.

**Solution:**

For the B-tree and the B*-tree, please derive analytic formulae which enable the determination of the upper and lower bound for the height \( h \) of the tree when \( k \), \( k^* \), and \( n \) (number of elements in the tree) are given.

**Upper and lower bound for the height of the B-tree**