Homework Assignment 4

Reading Assignment: Rosen, Sections 1.3, 1.4, 1.5

I. 1. List all the members of the following sets
   A. \{ x \mid x \text{ is an integer such that } x^2 = 1 \}
   B. \{ x \mid x \text{ is a real number such that } x^3 = 27 \}
   C. \{ x \mid x \text{ is a positive odd integer less than 16 }\}
   D. \{ x \mid x \text{ is a positive integer so that } x \text{ is divisible by 3 and } x < 20 \}

2. What is the cardinality of the following sets?
   A. \emptyset
   B. \{ \emptyset, a \}
   C. \{ \emptyset, \{\emptyset\}, \{\} \}
   D. \{ \}
   E. \{ \}, \{ \{ \emptyset \} \}

3. Give the Power set of the Following sets
   A. \{ \}
   B. \{ \emptyset \}
   C. \{ \emptyset, a \}
   D. \{ a \}

4. Determine if the following sets are Power sets.
   A. \{ \emptyset, \{b\}, \{c\}, \{ b, c \} \}
   B. \{ \emptyset, \{ b \}, \{ \emptyset, b \} \}
   C. \{ \emptyset, \{ \emptyset \} \}
   D. \emptyset

5. Let A = \{a, b, d\} and B = \{1, 2\} Find:
   A. A \times B
   B. B \times A

6. Let A=\{a,b,c,d,e\}, B=\{a,b,c,d,e,f,g,h\} Find:
   A. A \cap B
   B. A \cup B
   C. A-B
   D. B-A

7. Let A=\{0,2,4,6,8,10\}, B=\{0,1,2,3,4,5,6\}, C=\{4,5,6,7,8,9,10\} Find:
   A. A \cap B \cap C
   B. A \cup B \cup C
   C. (A \cup B) \cap C
   D. (A \cap B) \cup C
   E. A \oplus B
I. Using predicate calculus, prove the following:
Given: All people walk or drive.
All people who walk wear shoes.
Prove: All people drive or wear shoes.
Universe of Discourse: People
In your solution, let \( W(x) = \text{person } x \text{ walks}, D(x) = \text{person } x \text{ drives}, \) and \( S(x) = \text{person } x \text{ wears shoes}. \) Don’t forget to use quantifiers.

III. a. Use a truth table to prove the following:
\[ \forall x \ P(x) \land \exists x \ Q(x) \Rightarrow \exists x \ (P(x) \land Q(x)) \]
b. Use the \textbf{contrapositive rule} and \textbf{DeMorgan’s laws} to reduce this statement to the statement in part a.
\[ \forall x \ (P(x) \lor Q(x)) \Rightarrow \exists x \ P(x) \lor \forall x Q(x) \]

IV. Fill in the truth tables for the following predicates.

<table>
<thead>
<tr>
<th>Quantifiers</th>
<th>( P(x,y) )</th>
<th>( x \in {0,1}, y \in {0,1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall x \forall y )</td>
<td>( (x \lor y) \land \neg(x \land y) )</td>
<td></td>
</tr>
<tr>
<td>( \exists x \exists y )</td>
<td></td>
<td></td>
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<tr>
<td>( \forall x \exists y )</td>
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<tr>
<td>( \exists y \forall x )</td>
<td>[if true ( y = \ldots )]</td>
<td></td>
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