Skeleton Based As-Rigid-As-Possible Volume Modeling

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Introduction

 As-rigid-as-possible (ARAP) shape modeling is a popular technique to obtain natural deformations. There have been many excellent methods.



Olga Sorkine, et al,: Laplacian Surface Editing. SGP2004

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- We are interested in the volume preservation.



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- We are interested in the volume preservation.
- VGL is a good approach. However, we do not want to break the manifoldness of ARAP surface modeling or sacrificing the speed.



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- We do it by leveraging the skeleton information.



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Linear LSE Rotation and edge length constraints Skeleton and volume constraints

Linear LSE with C^0 continuity

 Laplacian coordinates represent each point as the weighted difference between such point and its neighborhoods.



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- Given original coordinates V, the connectivity, and m control points, the reconstructed object V' can be obtained by minimizing: $\|LV' - \delta\|_2^2 + \sum_{i=1}^m \|v'_{c_i} - v_{c_i}\|_2^2$



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 ||UV' = δ||² + Σ^m ||y' = y ||²

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Rotation and edge length constraints

• Iterate two steps to recover rotations:

Olga Sorkine and Marc Alexa: As-Rigid-As-Possible Surface Modeling. Eurographics SGP2007 Olga Sorkine: Least-Squares Rigid Motion Using SVD

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Rotation and edge length constraints

- Iterate two steps to recover rotations:
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- Robustness, simplicity, efficiency.



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Rotation and edge length constraints

- Iterate two steps to recover rotations:
- Step1: Initial guess from solving naive LSE.
- Step2: Find optimal rotations, then update the linear system (edge length preserving).
- Robustness, simplicity, efficiency.
- However, there is no volume preserving constraint.



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Skeleton and volume constraints

• Use volumetric mesh? Manifoldness, computational complexity, etc.

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Skeleton and volume constraints

- Use volumetric mesh? Manifoldness, computational complexity, etc.
- Use both the skeleton and edge length constraint to roughly preserve the volume, without breaking the manifoldness of ARAP or increasing the computation complexity.



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• One-way coupling property.



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Mesh editing framework



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Mesh editing framework



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Mesh editing framework

- Manually define the skeleton.
- Evenly generate skeleton points, and connect them with surface vertices automatically.



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Mesh editing framework

- Manually define the skeleton.
- Evenly generate skeleton points, and connect them with surface vertices automatically.
- Manually select anchor points (bottom) and control points (top).

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	Remesh
	Deformation
	Preprocess Anchor Pa Cantral Ph Done Cantol A
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Skeleton with ARAP	
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Introduction Linear LSE Algorithms Rotation and edge length constr Experiments Skeleton and volume constraints

Mesh editing framework

- Manually define the skeleton.
- Evenly generate skeleton points, and connect them with surface vertices automatically.
- Manually select anchor points (bottom) and control points (top).
- Interactively deform the shape.



Experimental settings

- The C++ implementation was run on a Intel Core2 Quad 2.40GHz CPU with 8G RAM.
- We compare the linear LSE, ARAP surface modeling and our method.
- We tested on the cactus model (620 vertices, 1, 236 polygons) and the horse model (2, 482 vertices, 4, 960 polygons).
- The relative root mean square errors of edge lengths and volume magnitudes are reported.

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Experimental settings Results

Results



Model	RRMS-E	RE-V	Times
(b)	0.126	0.453	0.017
(c)	0.074	0.131	0.024
(d)	0.075	0.056	0.025

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Experimental settings Results

Results



Model	RRMS-E	RE-V	Times
(b)	0.068	0.356	0.117
(c)	0.040	0.125	0.121

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Experimental settings Results

Conclusions

- We proposed an approach to approximately preserve the volume without breaking the manifoldness of traditional ARAP or increasing the computational complexity.
- Our method is easy-to-implement and may be useful to systems relying on ARAP techniques.
- Limitations: Skeleton generation; complex skeletons; self intersection.

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Experimental settings Results

Thanks for listening.

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