Coordinating the Motions of Multiple Robots with Specified Trajectories

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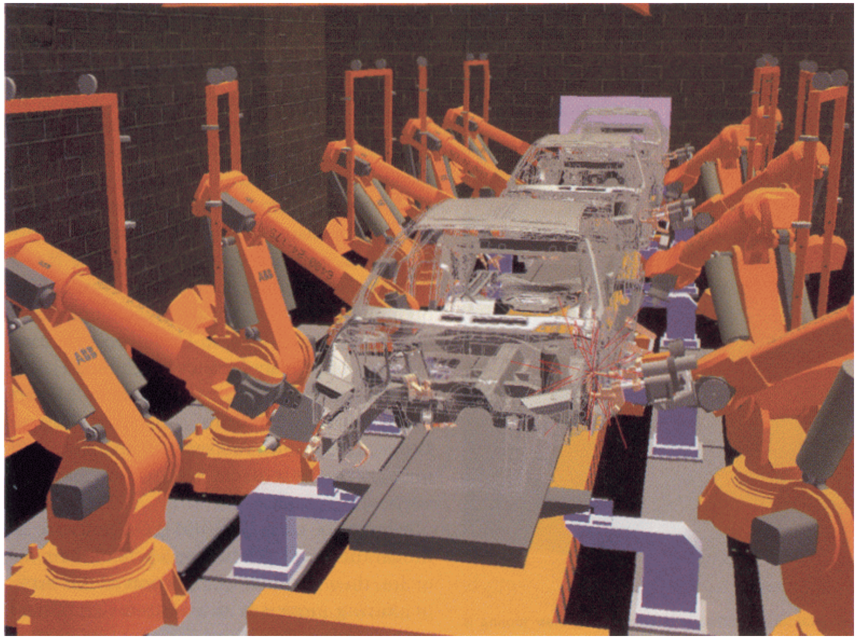
Trajectory Coordination Problem

- Given: Multiple robots with specified trajectories
- Find: Minimum time collision-free coordination schedule

20 Robot Mayhem: Before Coordination

Motivation

- Welding and painting robots in automotive industry



- AGVs in factories, harbors, and airports

Approach

- Start time trajectory modification
- Collision zones: geometry and timing
- Two robot coordination:MILP formulation
- Multiple robot coordination:MILP formulation, complexity

Related Work

- **Motion planning for multiple robots:** Hopcroft, Schwartz, Sharir (1984); Erdmann and Lozano-Perez (1987); Barraquand and Latombe (1991); Svestka and Overmars (1996); Aronov et al. (1999); Bicchi and Pallottino (2001); Sanchez and Latombe (2002)
- **Single robot among moving obstacles:** Reif and Sharir (1985); Kant and Zucker (1986)
- **Path coordination:** O'Donnell and Lozano-Perez (1989); LaValle and Hutchinson (1998); Leroy, Laumond, and Simeon (1999)

Related Work (cont.)

- **Trajectory coordination of two robots:** Lee and Lee (1987); Bien and Lee (1992); Chang, Chung, and Lee (1994); Shin and Zheng (1992)
- **Job shop scheduling:** Garey, Johnson, and Sethi (1976); Lawler et al. (1993); Sahni and Cho (1979); Goyal and Sriskandarajah (1988)

Paths and Trajectories

- **Path:** (γ) Geometric specification of a curve in configuration space
- **Trajectory:** (τ) A path together with time derivatives that provide the velocity profile

Modifying Start Times

- Use trajectories that give the desired velocity profiles by changing start times

$$\tau_i'(t) = \begin{cases} \tau_i(t - t_i^{start}) & : \quad t \geq t_i^{start} \\ 0 & : \quad t < t_i^{start} \end{cases}$$

- t_i^{start} : start time for robot A_i

Trajectory Coordination Problem

- **Given:** A set of robots $\{A_1, \ldots, A_n\}$ with specified trajectories
- **Find:** Start times such that completion time for set of robots is minimized and no collisions occur

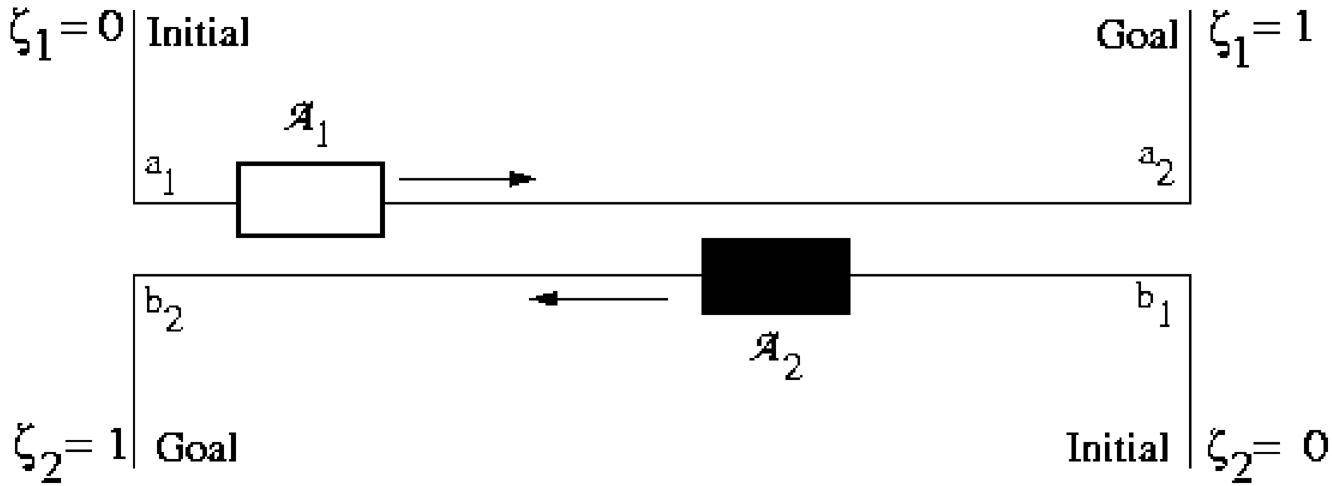
Assumptions

- Robots are the only moving objects
- No obstacles along paths
- Robot start and goal configurations are collision-free
- Each robot moves monotonically along its path
- Collisions sampled at sufficiently fine resolution

Collision Zones: Geometry

- A collision zone for robot A_i with robot A_j is a contiguous interval of path positions ζ_i such that

$$\mathcal{A}_i(\gamma_i(\zeta_i)) \cap \mathcal{A}_j(\gamma_j(\zeta_j)) \neq \emptyset$$



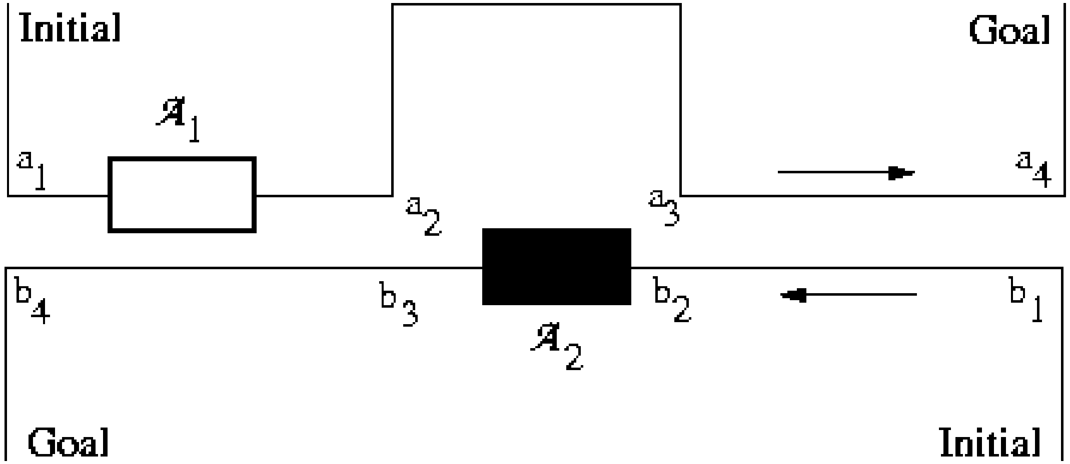
- The set of collision zones for A_i with A_j is PB_{ij}

$$\mathcal{PB}_{ij} = \{[\zeta_{is}^k, \zeta_{if}^k]\}$$

Collision Zone Pairs

- The set of collision zone pairs, PI $_{ij}$

$$\mathcal{PI}_{ij} = \{< [\zeta^k_{is}, \zeta^k_{if}], [\zeta^k_{js}, \zeta^k_{jf}] >\}.$$



- Example: PI $_{12}$ is $\{<[a_1,a_2],[b_3,b_4]>, <[a_3,a_4],[b_1,b_2]>\}$

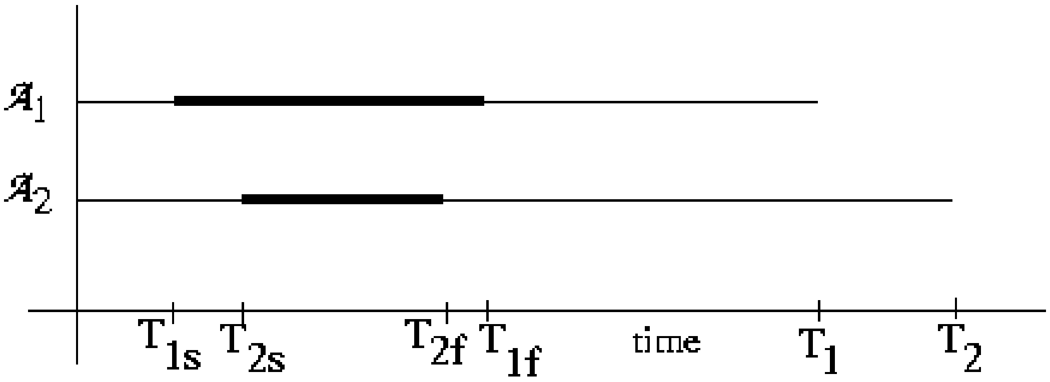
Collision Zones: Timing

- Identifying the times when collisions can occur is critical for scheduling the robots
- TB_{ij} : set of times A_i could collide with A_j

$$
\begin{aligned}
\mathcal{TB}_{ij}(\tau_i) &= \{t \mid \mathcal{A}_i(\gamma_i(\tau_i(t))) \cap \mathcal{A}_j(\gamma_j(\zeta_j)) \neq \emptyset, \\
&\qquad \text{for some } \zeta_j \in [0,1], i \neq j\} \\
&= \tau_i^{-1}(\mathcal{PB}_{ij}) \\
&= \{[\tau_i^{-1}(\zeta_{is}^k), \tau_i^{-1}(\zeta_{if}^k)]\}.
\end{aligned}
$$

Collision-time Interval Pairs

- Set of all collision-time interval pairs for A_i and A_j, $\mathcal{CI}_{ij} = \{< [T^k_{is}, T^k_{if}], [T^k_{js}, T^k_{jf}] >\}$



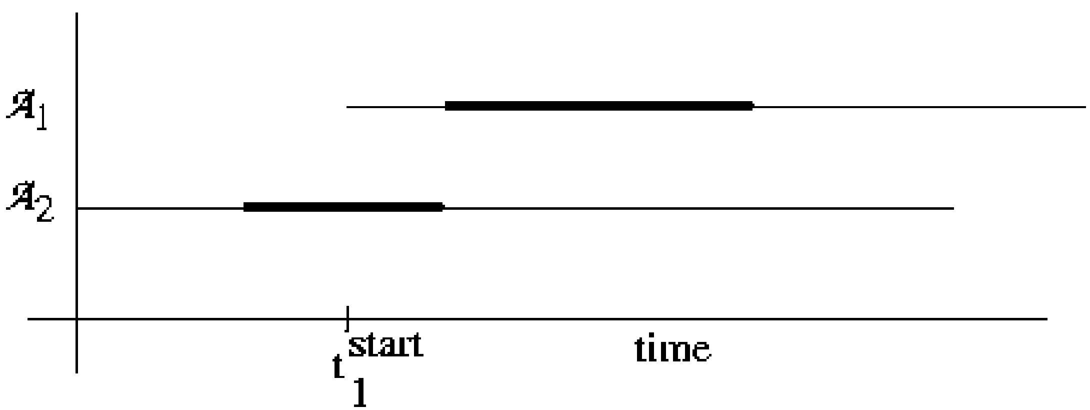
T_{is}^k and T_{if}^k are start and finish times of kth collision-time interval,

T_i is completion time for robot A_i

Sufficient Conditions for Collision-free Scheduling

- To avoid collisions between A_i and A_j, sufficient to ensure A_i and A_j are not simultaneously in a collision zone pair
- No collision can occur if I^k_i Å I^k_j = ;

where $I_i^k = [T_{is}^k + t_i^{start}, T_{if}^k + t_i^{start}]$



Optimization Problem II

- **Given:** A set of robots with specified trajectories
- **Find:** Start times to minimize completion time so there is no overlap of paired collision-time intervals
- Note: relaxed version of Trajectory Coordination Problem

Two Robot Case

- Assume robots have single collision region
- Optimization problem is:

$$\text{Minimize } \max\{t_i^{start} + T_i, t_j^{start} + T_j\}$$

subject to

$$t_i^{start} + T_{if} < t_j^{start} + T_{js} \text{ or}$$

$$t_i^{start} + T_{is} > t_j^{start} + T_{jf}$$

$$t_i^{start} \geq 0$$

$$t_j^{start} \geq 0$$

Two Robots: MILP Formulation

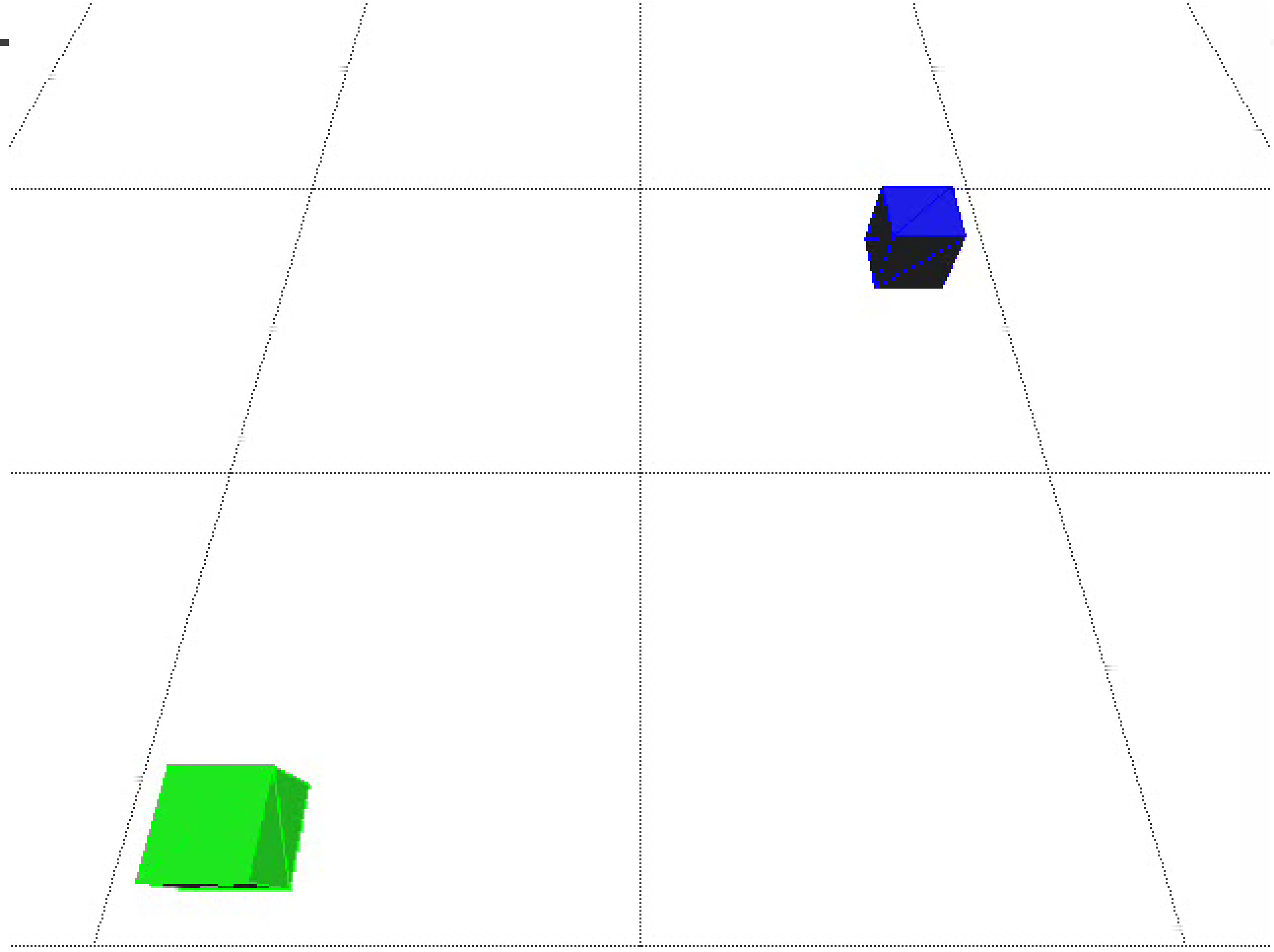
- Mixed Integer Linear Program (MILP)

Minimize $t_{complete}$
subject to

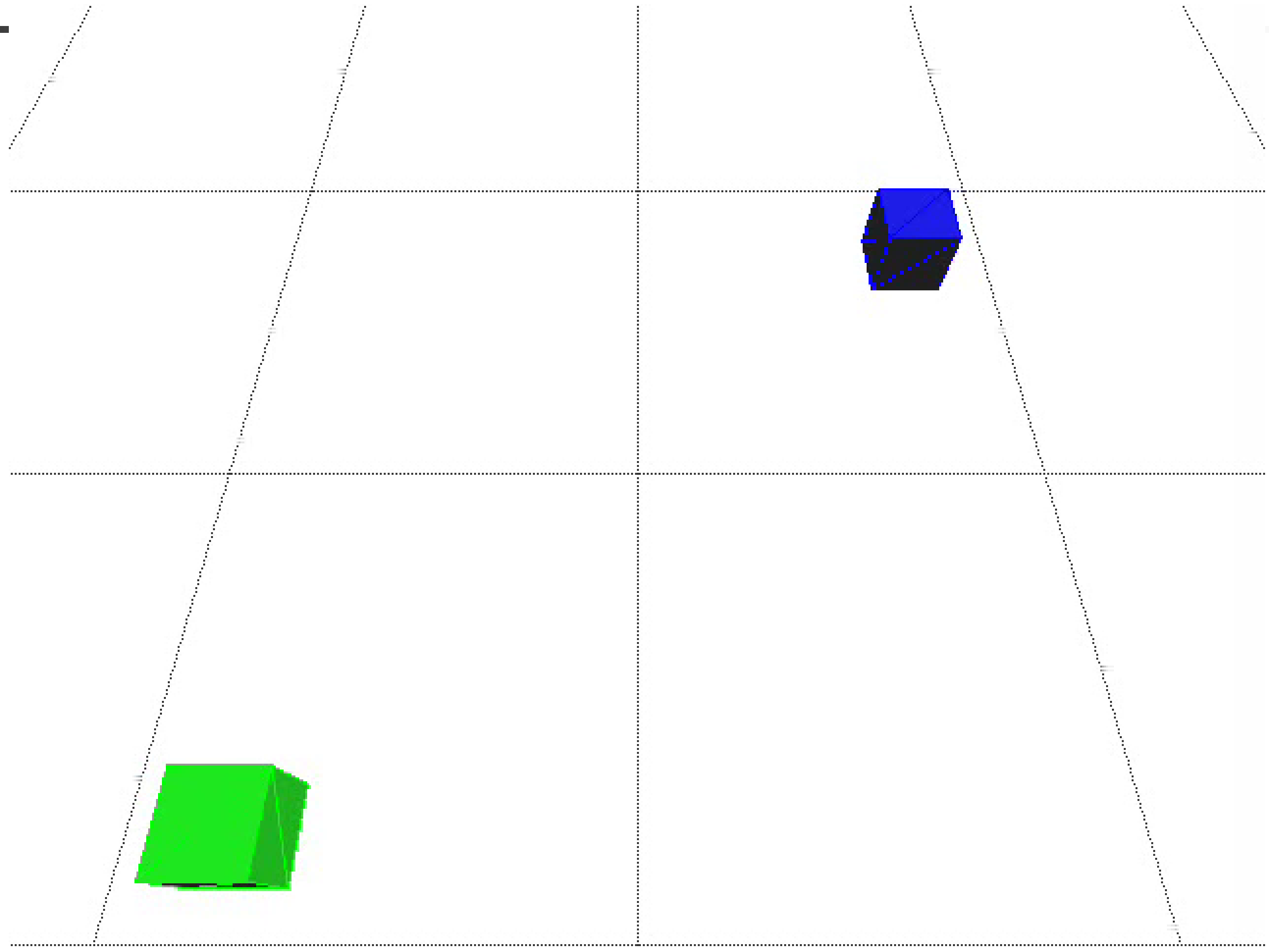
$$t_{complete} - t_i^{start} - T_i \geq 0$$
$$t_{complete} - t_j^{start} - T_j \geq 0$$
$$t_i^{start} + T_{if} - t_j^{start} - T_{js} - M\delta_{ij} \leq 0$$
$$t_j^{start} + T_{jf} - t_i^{start} - T_{is} - M(1 - \delta_{ij}) \leq 0$$
$$t_i^{start} \geq 0$$
$$t_j^{start} \geq 0$$
$$\delta_{ij} \in \{0, 1\}$$

δ_{ij} is 0 if $\mathcal{A}_i$ goes first in collision zone, and is 1 if $\mathcal{A}_j$ goes first

Example: Before Coordination

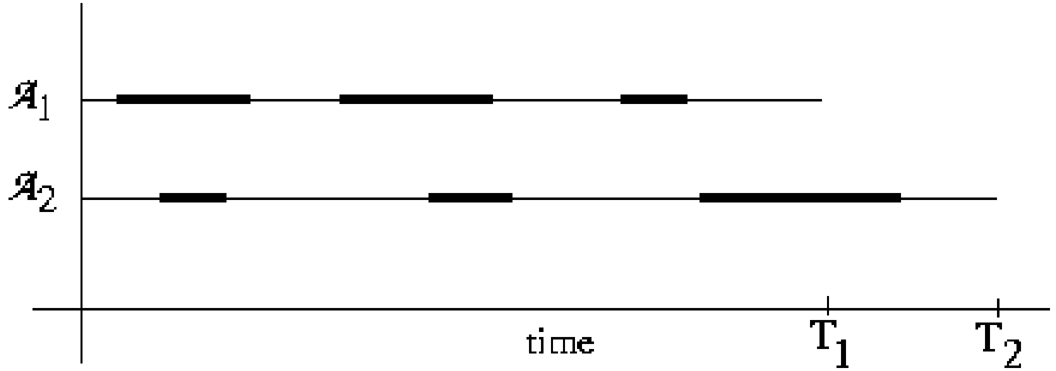


Example: After Coordination

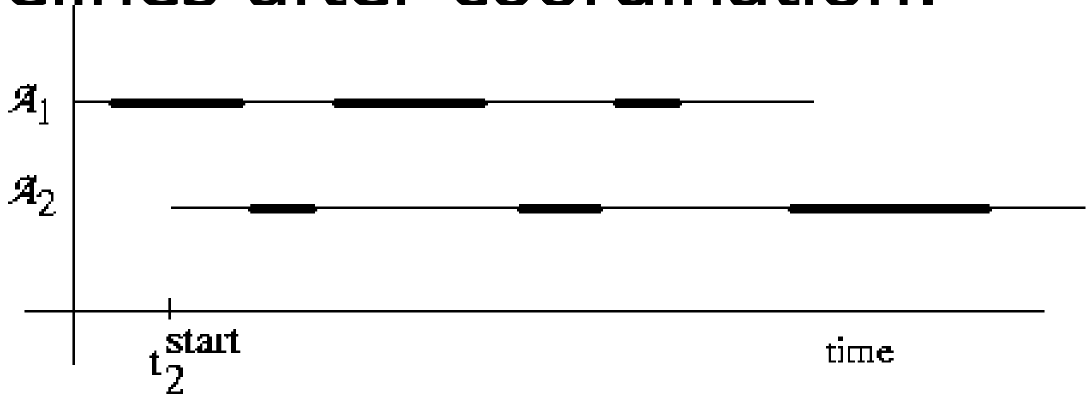


Two Robots:Multiple Collisions

- Timelines before coordination:

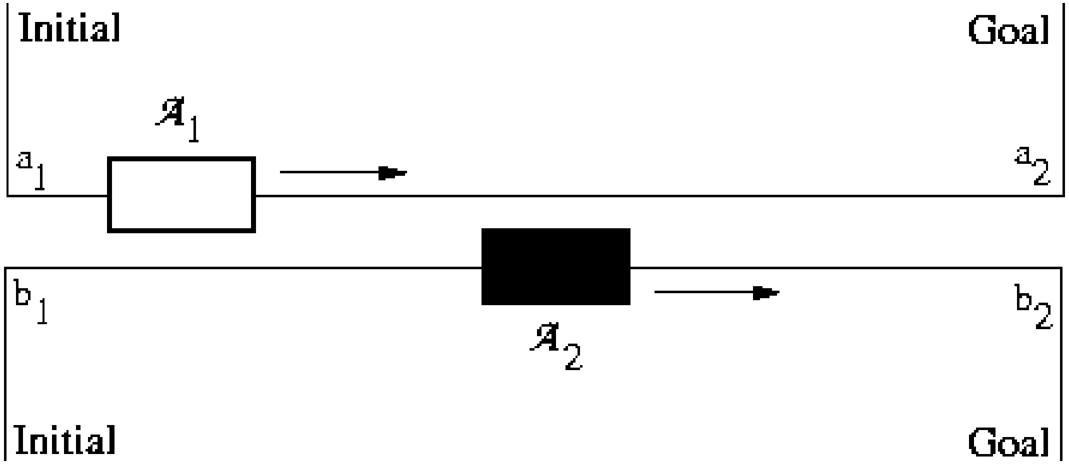


- Timelines after coordination:



Can We Do Better?

- Requiring that robots not simultaneously be in shared collision zone is conservative
- Consider two robots moving in the same direction in a collision region



- Let robots play "follow the leader"

Follow the Leader

- Compute min lead time T_{ijk}^{lead} , for every collision zone pair, by bisection search

- Follow the leader constraints are:

$t_i^{start} + T_{is}^k + T_{ijk}^{lead} < t_j^{start} + T_{js}^k$ when $\mathcal{A}_i$ leads through kth collision zone,

or

$t_j^{start} + T_{js}^k + T_{jik}^{lead} < t_i^{start} + T_{is}^k$ when $\mathcal{A}_j$ leads through kth collision zone.

Follow the Leader Formulation: Two Robots

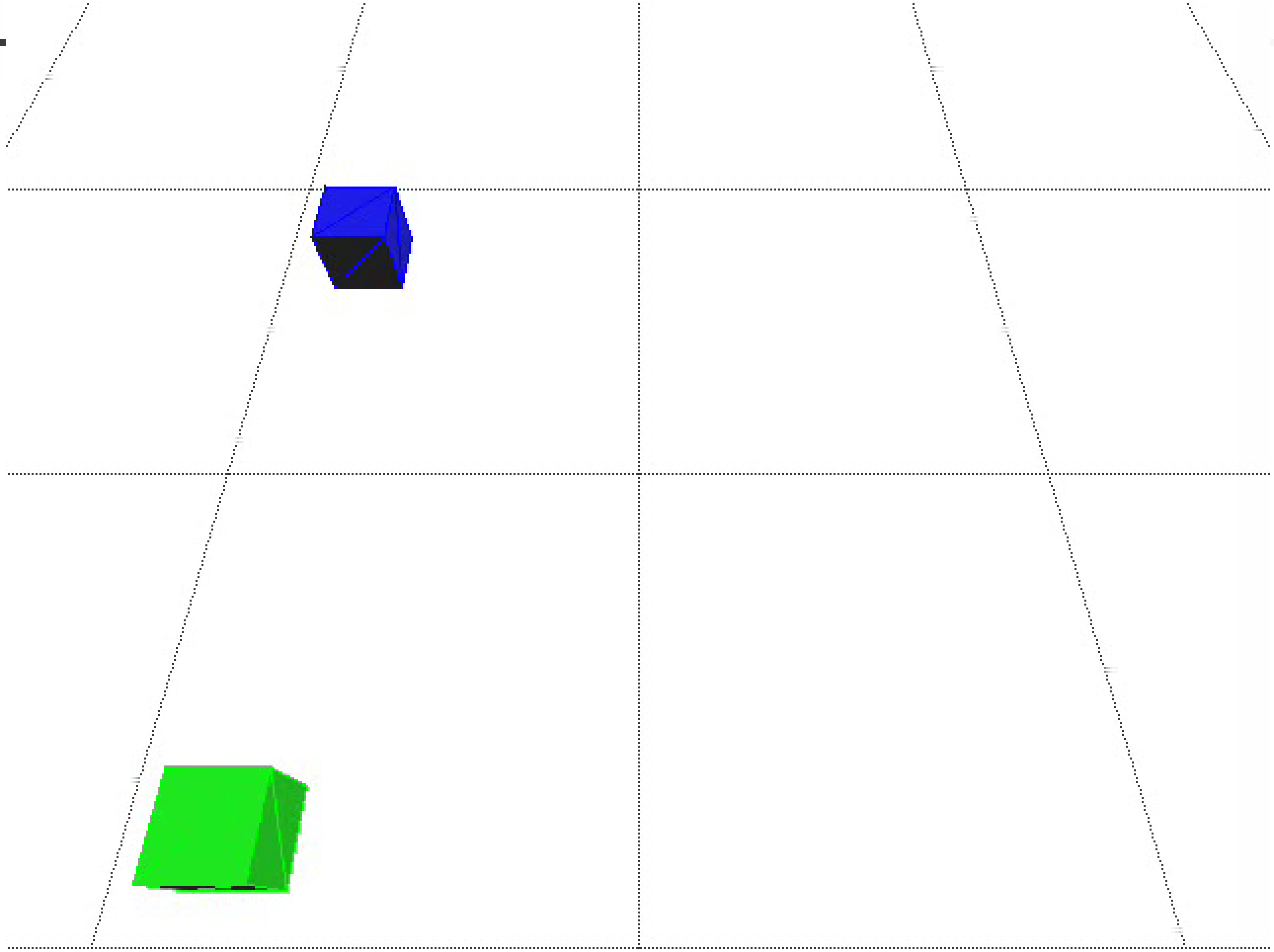
- MILP formulation with lead times

Minimize $t_{complete}$
subject to

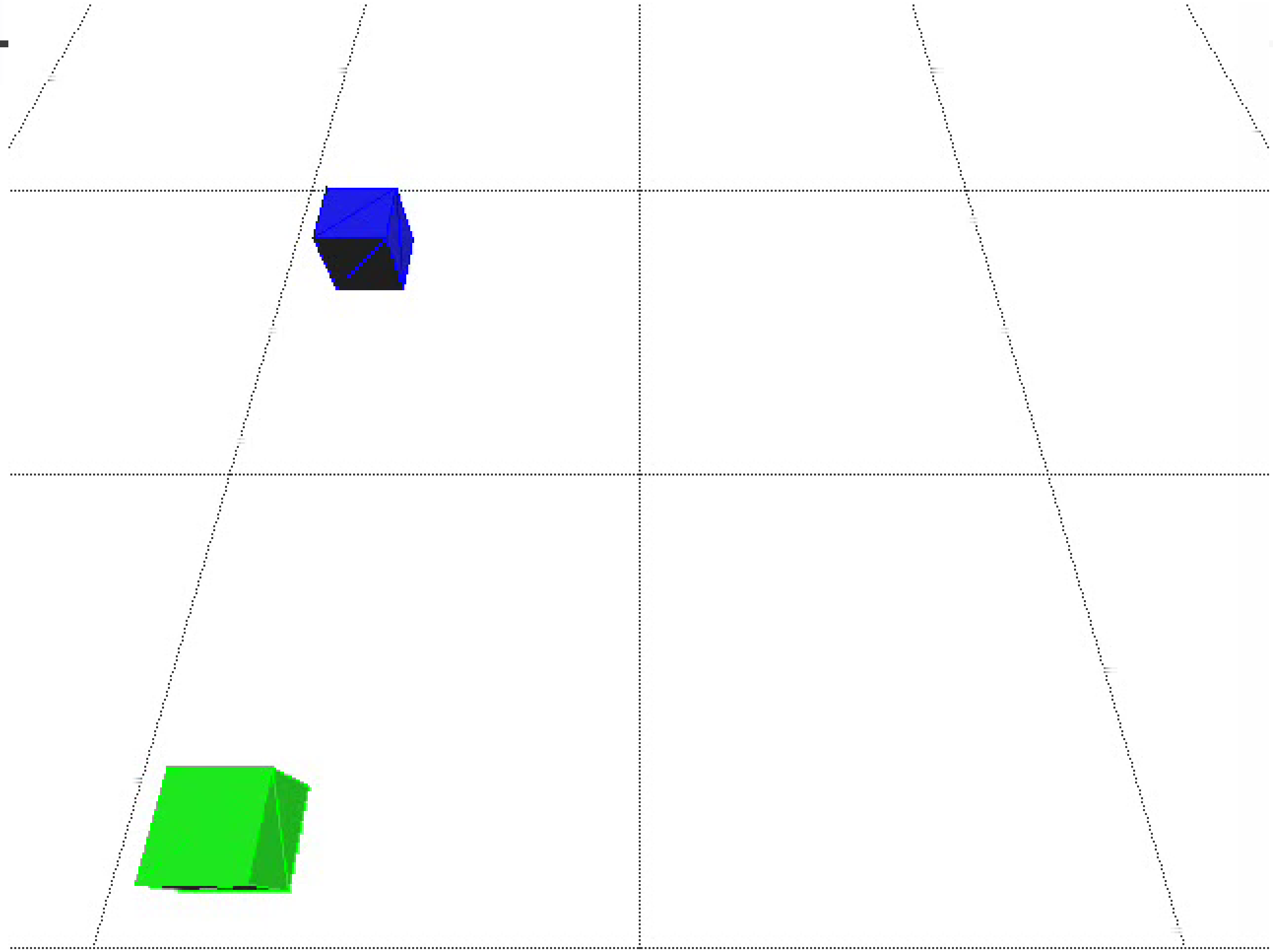
$$
\begin{array}{ll}
t_{complete} - t_i^{start} - T_i \geq 0 & \\
t_{complete} - t_j^{start} - T_j \geq 0 & \\
t_i^{start} + T_{ie}^k - t_j^{start} - T_{js}^k - M\delta_{ijk} \leq 0 & 1 \leq k \leq N_{ij} \\
t_j^{start} + T_{je}^k - t_i^{start} - T_{is}^k - M(1-\delta_{ijk}) \leq 0 & 1 \leq k \leq N_{ij} \\
\delta_{ijk} \in \{0,1\} & 1 \leq k \leq N_{ij} \\
t_i^{start} \geq 0 & \\
t_j^{start} \geq 0 & .
\end{array}
$$

where $T_{ie}^k = T_{is}^k + T_{ijk}^{lead}$

Example: Before Coordination



Example: After Coordination



MILP Formulation for Trajectory Coordination Prob.

- Gives optimal solution for multiple robots

Minimize $t_{complete}$
subject to

$$
\begin{array}{ll}
t_{complete} - t_i^{start} - T_i \geq 0 & 1 \leq i \leq N_{robots} \\
t_i^{start} + T_{ie}^k - t_j^{start} - T_{js}^k - M\delta_{ijk} \leq 0 & 1 \leq k \leq N_{ij} \\
t_j^{start} + T_{je}^k - t_i^{start} - T_{is}^k - M(1 - \delta_{ijk}) \leq 0 & 1 \leq i < j \leq N_{robots} \\
\delta_{ijk} \in \{0, 1\} & 1 \leq k \leq N_{ij} \\
t_i^{start} \geq 0 & 1 \leq i \leq N_{robots}.
\end{array}
$$

Complexity of Trajectory Coordination

- The trajectory coordination problem for multiple robots is NP-hard:

  Reduction from No-wait Job Shop Scheduling problem (Sahni and Cho 1979)

20 Robot Mayhem:
A Tragedy Averted

6 Articulated Robots: Before Coordination

Implementation

- C++, PQP (Larsen et al. 2000), CPLEX

#robots	#collision zone pairs	collision detection time (secs)	MILP time (secs)
5	10	2.4	0.02
10	27	9.8	0.11
15	65	23.4	0.53
20	79	36.8	1.83

- Running time depends critically on number of collision zone pairs

Conclusions

- Trajectory coordination of multiple robots, when start times can be varied, achieved using MILP formulation
- Complexity depends on number of potential collisions and number of robots, relatively independent of DOF
- Trajectory coordination is NP-hard
- Implemented planner demonstrated on 20 robots

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Future Work

- Velocity tuning to modify trajectories, reduce completion time
- Velocity coordination: Given paths, generate continuous velocity profiles
- Approximation algorithms for trajectory coordination
- Incorporating timing uncertainties
- Choreography of animation characters