Optimal design of parallel manipulators based on their dynamic performance

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- Quantification of dynamic performance of a parallel manipulator
- Optimisation of dynamic performance, using the above

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- Reduced mass of the manipulator
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- Generalised Inertia Ellipsoid (Asada *et al*, 1983)
- Dynamic manipulability (Yoshikawa *et al*, 1985)
- The concept of dynamic isotropy (Ma *et al*, 1990)
- The dynamic capability equations for non-homogeneous task space (Bowling *et al*, 2000)
- Dynamic performance indices for 3-DoF parallel manipulators (Gregorio *et al*, 2002)
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Research issues

- Intrinsic vs. extrinsic
- Incorporating three disparate objects – $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{G}$
- Local vs. global: restriction to feasible regions
- Dimensional vs. non-dimensional indices
- Homogeneous vs. non-homogeneous task space
- Computational complexities
- Actaul vs. theoretical link model

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Present work: a new formulation of the problem

New contributions:

- Combination of dimensional and non-dimensional indices
- Restriction to the safe working zone (SWZ): extension of local indices to global
- Intrinsic formulation motivated by physical intuitions, but validated empirically
- Applicable to non-homogeneous task space

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Physical motivation: analogy with a particle

- A particle of constant mass m, moving in $\mathbb{R}^n$, has the simplest possible *inertia matrix*:

$$\boldsymbol{M} = m\boldsymbol{I}_{n\times n},\ m \in \mathbb{R}^+$$

- The n DoF are completely *decoupled*
- The inertia is identical in *all* directions, i.e., the inertia is *isotropic*
- If the m above is *small*, then the system responds fast

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Local indices : intrinsic measures for n-DOF system

- Index μ_1 (dynamic isotropy index) is formulated to measure isotropy of mass matrix ($\boldsymbol{M}$) as,

$$\mu_1(\boldsymbol{M}) = \frac{n^2}{\kappa(\boldsymbol{M})}, \quad 0 \le \mu_1(\boldsymbol{M}) \le 1$$

$$\text{where,} \quad \kappa(\boldsymbol{M}) = \left(\frac{1}{\lambda_1} + \cdots + \frac{1}{\lambda_n}\right)(\lambda_1 + \cdots + \lambda_n).$$

- Index μ_2 (inertia index) is formulated to measure the maximum inertia using $\boldsymbol{M}$ as,

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Local indices are extended to global indices over a subset of the workspace, such as the SWZ:

- Global isotropy index:

$$\bar{\mu}_1(\boldsymbol{M}) = \frac{\int_V \mu_1(\boldsymbol{M})dv}{\int_V dv}$$

- Global inertia index:

$$\bar{\mu}_2(\boldsymbol{M}) = \max_V(\mu_2(\boldsymbol{M})) = \max_V\left(\max_i\{\lambda_i(\boldsymbol{M})\}\right)$$

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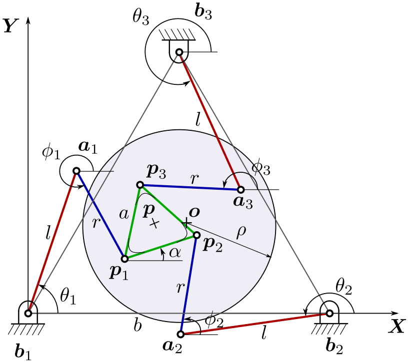
Formulation of the optimisation problem

$$\text{Minimise} \begin{cases} f_1(\boldsymbol{x}) = -\bar{\mu}_1 \\ f_2(\boldsymbol{x}) = \bar{\mu}_2 \end{cases}$$

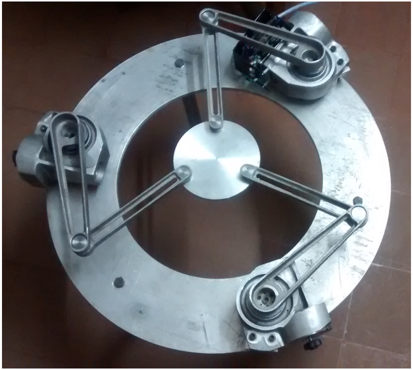
$$\text{subject to: } g_i(\boldsymbol{x}) \leq 0,$$

$$x_j \in [a_j, b_j], \quad j = 1, \ldots, m.$$

3-RRR planar parallel manipulator (PPM)

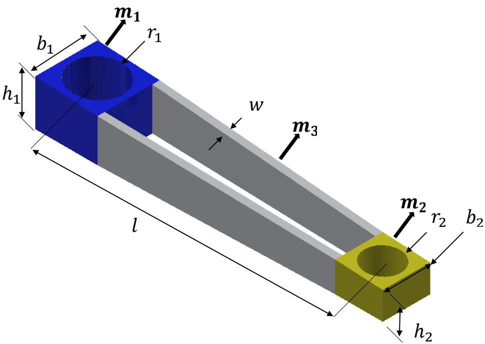


Kinematic details of 3-RRR planar parallel manipulator (PPM)

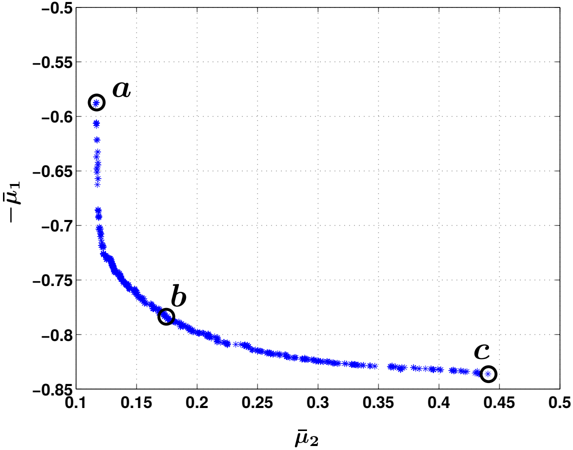


Commercial prototype developed at Systemantics India Pvt. Ltd.

Link modelling



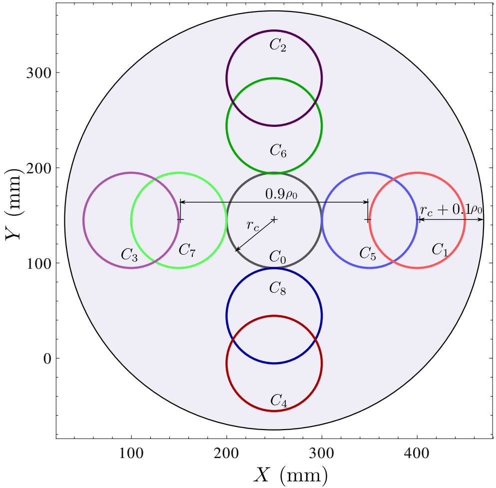
Results: Pareto front



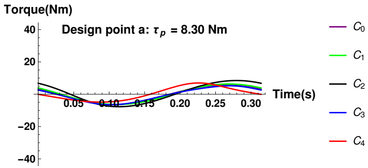
Comparison of designs from Pareto plot and existing design

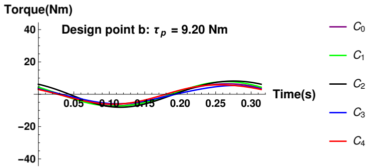
Design point	$\bar{\mu}_1$	$\bar{\mu}_2$ (kg-m^2)	τ_p(Nm) $u = 1$ m/s	τ_p(Nm) $u = 1.5$ m/s
a	**0.58**	**0.12**	**8.30**	**19.08**
b	0.78	0.17	9.20	19.92
c	0.84	0.43	12.33	27.74
Existing	0.48	1.19	16.48	37.67

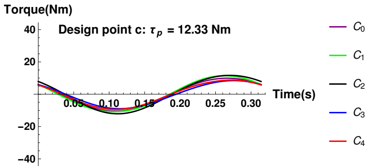
Validation via inverse dynamic simulations

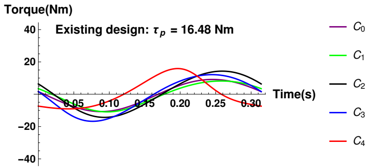


Torque plots: $u = 1.0$ m/s

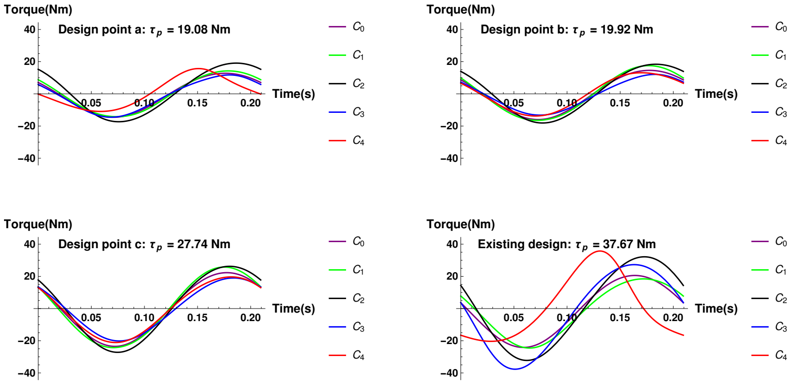








Torque plots: $u = 1.5$ m/s



Discussions: advantages

- Intrinsic indices are used for global enhancement of performance, which seem to agree with extrinsic results
- Dimensional inhomogenity is taken care of, to a large extent
- No further validation is required, as the analysis is confined to the SWZ

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- Considers only the inertia terms, and not the potential ones
- May suffer from dimensional inhomogenity, in cases where actuators are of mixed type

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Thank you for your attention!

Questions/comments?

Sample results: design point a

Design	Existing Design	Design point: a
l (mm)	500	307
r (mm)	500	500
a (mm)	150	136
b (mm)	1000	938
α_{mid} (deg)	68	62