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Dynamics and Optimal Feet Force Distributions of a Realistic Four-legged Robot

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ABSTRACT

This paper presents a detailed dynamic modeling of realistic four-legged robot. The direct and inverse kinematic analysis for each leg has been considered in order to develop an overall kinematic model of the robot, when it follows a straight path. This study also aims to estimate optimal feet force distributions of the said robot, which is necessary for its real-time control. Three different approaches namely, minimization of norm of feet forces (approach 1), minimization of norm of joint torques (approach 2) and minimization of norm of joint power (approach 3) have been developed. Simulation result shows that approach 3 is more energy efficient foot force formulation than other two approaches. Lagrange-Euler formulation has been utilized to determine the joint torques. The developed dynamic models have been examined through computer simulation of continuous gait of the four-legged robot.

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1. INTRODUCTION

Recently, many studies have been carried out on multi-legged walking robots because walking robots offer better mobility. Multi-legged robot has the advantage over the wheeled robot as it used the isolated point to support the trunk not the continuous path that is needed by wheeled robot. It can get steady walking on uneven terrain and avoid the obstacle and can get omni-directional motion by keeping the terrain intact. It can climb stair and rugged mountain, navigate over planet surface. All of the advantages make the multi-legged robot become an important and active area of research in the field of mobile robotics. Because the four-legged robot has more carrying capacity and good stability than the biped robot, and has the more simple structure than the six-legged robot and eight-legged robot. So, the quadruped robot arouses extensive attention. Design of the legged-robot is a complicated problem in applied mechanics and robotics. It needs the solving of many interrelated problems like kinematics, gait planning, trajectory generation, dynamics, control etc. In order to develop efficient control algorithm of robots, it is important to have good models describing the kinematic and dynamic behaviors of the complex multi-legged robotic mechanism. In this context, Koo and Yoon [1] obtained a mathematical model for four-legged walking robot to investigate the dynamics after considering all the inertial effects in the system. Pfeiffer et al. [2] investigated the dynamics of a stick insect walking on flat terrain. Freeman and Orin [3] developed an efficient dynamic simulation of a quadruped robot using a decoupled tree-structure approach. A dynamic model of four-legged walking robot was derived by Lin and Song [4] to study the dynamic stability and energy efficiency during walking.

To control the motion of the robot, the trunk body motion controller calculates the resultant control wrench (i.e., force and moment), that should be applied to the robot's body by its supporting legs. Therefore, one of the important issues of a legged robot's active force control is a successful distribution of its body force to the feet. For a statically stable multi-legged robot, at least three legs should be on the ground at any instant. If a three-dimensional reaction force vector is considered on each ground leg, the foot force distribution problem becomes indeterminate during the walking because of the closed chain system. Multiple solutions might exist, which can satisfy the force-moment balance criteria. In this connection, work of Howard et al. [5], Gorinevsky and Shneider [6], Jiang et al. [7], Barreto et al. [8] and Gonzalez de Santos et al. [9] are worth mentioning.

Zhou et al. [10] proposed a new force distribution method called Friction Constraint Method (FriCoM) to evaluate reaction forces at each ground leg by considering the friction constraints during the walking of a four-legged robot. Results of the FriCoM were compared with those obtained by the pseudo-inverse method [7] and an incremental method [11]. The FriCoM was found to be more practical compared to the pseudo-inverse method. Moreover, it was seen to be computationally faster than the incremental method and thus, found to be suitable for real-time control of quadruped robots. Unfortunately, it did not consider any locomotion performance objectives, such as minimization of foot force components, minimization of joint torques or minimization of energy consumption etc. The minimization of energy consumption plays an important role in the locomotion of a multi-legged robot used for service applications. Marhefka and Orin [12] utilized quadratic programming to solve feet forces distributions in hexapod walking robots that minimizes the power consumption in DC motors. Kar et al. [13] used sequential quadratic programming method to determine energy optimal foot force and performed an analysis of energy efficiency with respect to structural parameters, interaction forces, friction coefficient and duty factor of wave gaits, based on a simplified model of six-legged robot. Kar et al. [13], and Lin and Song [14] considered instantaneous power to be the product of instantaneous joint torques and joint velocities. Erden and Leblebicioglu [15] utilized modified simplex method along with Lemke's Complementary pivoting algorithm to compute optimum foot force and torque distributions by considering a more practical locomotion performance objective, that is, minimization of energy dissipation. Although the above attempt could find the optimal values of feet forces of the multi-legged robot, they might not be suitable for real-time implementations because the used optimization techniques were iterative in nature. Moreover, due to inherent complexity of a realistic walking robot, it is not an easy task to include inertial terms in the modeling.

The most of the studies on walking dynamics were conducted with a simplified model of legs and body. However, in order to have a better understanding of walking and other important issues of walking, such as dynamic stability, energy efficiency and on-line control, kinematic and dynamic models based on a realistic walking robot design are necessary. Here, an attempt has been made to carry out kinematics, dynamics and optimal feet force distributions of a realistic four-legged robot.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

This section deals with mathematical formulation of the problem and explains the proposed methods to find optimal feet forces.

2.1. Kinematics and Foot Trajectory Analysis

The robot considered in this study (Figure 1) consists of a trunk body of rectangular cross-section and four legs, which are similar and symmetrically distributed on either sides of the trunk body. Each leg has three powered rotary joints with the typical articulated (RRR) configuration, i.e. the second and third joints' axes are parallel to each other and perpendicular to the first joint's axis. The three actuators are dc-servomotors with a permanent-magnet stator. The Denavit-Hartenberg (D-H) notations [16] have been used in kinematic modeling of each leg (refer to Figure 2).Table 1 shows four D-H parameters, namely link length (a_i), link twist (α_i), joint distance (d_i), and joint angle (θ_i), required to completely describe the leg mechanism.

Table 1. D-H parameters for three joint legs

Link no.	a_i	α_i	d_i	θ_i
1	a_1=0.085m	90°	0	θ_1
2	a_2=0.100m	0	0	θ_2
3	a_3=0.115m	0	0	θ_3

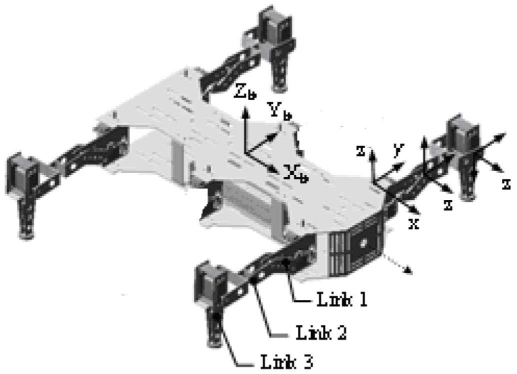

Figure 1. The model of four-legged robot

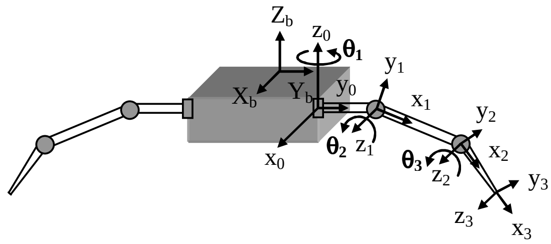

Figure 2. D-H representation of link frame

The homogeneous transformation matrix [17] describing the relative translation and rotation between i^{th} and $(i-1)^{th}$ coordinate systems is represented as follows:

$$^{i-1}\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, foot tip reference frame {3} can be expressed in the leg reference frame {0} as given below.

$$^{0}\mathbf{T}_3 = {}^{0}\mathbf{T}_1\,{}^{1}\mathbf{T}_2\,{}^{2}\mathbf{T}_3$$

$$^{0}T_3 = \begin{bmatrix} \cos\theta_1\cos(\theta_2+\theta_3) & -\cos\theta_1\sin(\theta_2+\theta_3) & \sin\theta_1 & \left(a_1+a_2\cos\theta_2+a_3\cos(\theta_2+\theta_3)\right)\cos\theta_1 \\ \sin\theta_1\cos(\theta_2+\theta_3) & -\sin\theta_1\sin(\theta_2+\theta_3) & -\cos\theta_1 & \left(a_1+a_2\cos\theta_2+a_3\cos(\theta_2+\theta_3)\right)\sin\theta_1 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & a_2\sin\theta_2+a_3\sin(\theta_2+\theta_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, the position of the robot's foot tip can be represented in its general form as given below.

$$[a_1+a_2\cos\theta_2+a_3\cos(\theta_2+\theta_3)]\cos\theta_1 = p_x\,, \quad (1)$$

$$[a_1+a_2\cos\theta_2+a_3\cos(\theta_2+\theta_3)]\sin\theta_1 = p_y\,; \quad (2)$$

$$a_2\sin\theta_2+a_3\sin(\theta_2+\theta_3) = p_z \quad (3)$$

By solving equations (1), (2) and (3), the joint angles: θ_1, θ_2 and θ_3 have been determined.

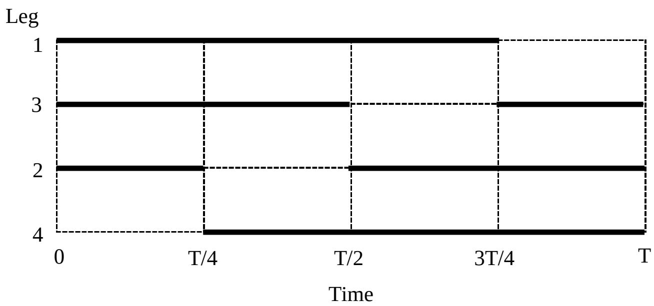

Figure 3. Gait diagram with duty factor 3/4

The robot moves straight forward at a constant velocity on flat surface with continuous gait (duty factor β=3/4) (refer to Figure 3). To ensure a smooth path to be followed, each joint trajectory followed by the swing leg joints is assumed to be governed by a fifth degree polynomial which is a function of time (t). The jth joint of a swing leg, that is, θ_j can be represented in fifth-order polynomial as follows:

$$\theta_j = c_{j0}+c_{j1}t+c_{j2}t^2+c_{j3}t^3+c_{j4}t^4+c_{j5}t^5+c_{j6}t^6 \tag{4}$$

where c_{j0}, c_{j1}, c_{j2}, c_{j3}, c_{j4}, c_{j5} and c_{j6} are the coefficients, whose values are determined using a set of boundary conditions defined over the swing phase and j=1, 2, 3 joints. The boundary conditions of joint angles at at initial, middle and final points, joint rates and joint accelerations at initial and final points of the trajectory are applied to find the seven coefficients for each joint. The joint rate and joint acceleration equations of each joint of a swing leg can be obtained using the following equations:

$$\dot{\theta}_j = c_{j1}+2c_{j2}t+3c_{j3}t^2+4c_{j4}t^3+5c_{j5}t^4+6c_{j6}t^5 \tag{5}$$

$$\ddot{\theta}_j = 2c_{j2}+6c_{j3}t+12c_{j4}t^2+20c_{j5}t^3+30c_{j6}t^4 \tag{6}$$

The joint rate and joint acceleration equations of for each leg during the support phase can be expressed as follows:

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^{-1}\dot{\mathbf{p}}\,, \tag{7}$$

$$\ddot{\boldsymbol{\theta}} = \mathbf{J}^{-1}(\ddot{\mathbf{p}} - \dot{\mathbf{J}}\dot{\boldsymbol{\theta}})\,, \tag{8}$$

where the position vector $\mathbf{p}=[p_x\ p_y\ p_z]^T$, $\boldsymbol{\theta}=[\theta_1\ \theta_2\ \theta_3]^T$ and $\mathbf{J}$ is the Jacobian matrix, which has been obtained as given below.

$$\mathbf{J} = \begin{bmatrix} -\left(a_1+a_2C\theta_2+a_3C(\theta_2+\theta_3)\right)S\theta_1 & -\left(a_2S\theta_2+a_3S(\theta_2+\theta_3)\right)C\theta_1 & -a_3S(\theta_2+\theta_3)C\theta_1 \\ \left(a_1+a_2C\theta_2+a_3C(\theta_2+\theta_3)\right)C\theta_1 & -\left(a_2S\theta_2+a_3S(\theta_2+\theta_3)\right)S\theta_1 & -a_3S(\theta_2+\theta_3)S\theta_1 \\ 0 & a_2C\theta_2+a_3C(\theta_2+\theta_3) & a_3C(\theta_2+\theta_3) \end{bmatrix} \tag{9}$$

2.2. Dynamics of Four-legged Robot

A four-legged robot is a complex linkage system, whose legs are connected to one another through the trunk body and also through the ground, and thus, forms closed kinematic chains. The basic problem of controlling these kinematic chains is their coordination. In addition to the local coordination problem, which involves control of the individual joints of a leg to achieve the desired tip control, there is a global coordination problem involving coordination among several chains of the multiple legs. The forces and moments propagate through the kinematic chains from one leg to another, and therefore, dynamic coupling exists. The equations of motion for such a complex mechanism with four legs, each of which has three degrees of freedom, are derived by applying Lagrangian dynamics formulation [17] together with Denavit-Hartenberg's link coordinate representation, and the derived relationships are given in the vector-matrix form as follows:

$$\boldsymbol{\tau}_i = \left[\mathbf{M}(\boldsymbol{\theta})\right]_i \left[\ddot{\boldsymbol{\theta}}\right]_i + \left[\mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\right]_i + \left[\mathbf{G}(\boldsymbol{\theta})\right]_i - \left[\mathbf{J}_i\right]^T \mathbf{F}_i\,, \tag{10}$$

where $\mathbf{M}(\boldsymbol{\theta})$ is the mass matrix of the leg, $\mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})$ is a vector of centrifugal and Coriolis terms, $\mathbf{G}(\boldsymbol{\theta})$ is a vector of gravity terms, $\boldsymbol{\tau}_i$ is the vector of joint torques, $\mathbf{J}_i$ is the Jacobian matrix and $\mathbf{F}_i$ is the vector of ground reaction forces of i^{th} foot. During the leg swing phase, there is no foot-terrain interaction, and $\mathbf{F_i}$ becomes equal to zero. However, during the support phase, ground contact exists and equation (10) becomes undetermined, which has to be solved using an optimization criterion, for example, optimal foot force distribution.

2.3. Optimum Feet Force Distributions

The problem of feet forces' distributions has been solved using three approaches, namely minimization of norm of feet forces (approach 1), minimization of norm of joint torques (approach 2) and minimization of norm of joint work (approach 3).

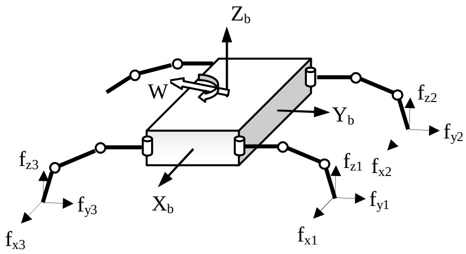

Figure 4. A schematic view showing feet contact forces acting on the robot

Approach 1: Minimization of Norm of Foot Forces

The contact between the foot tip and the ground is assumed to be hard-point contact with friction, which implies that the forces acting at the tip-point are restricted to three components, one normal and two tangential to the surface. Let us assume that $\mathbf{F}_i=[f_{xi}, f_{yi}, f_{zi}]^T$ is the ground-reaction force vector on foot i (where i=1,2, 3). The wrench $\mathbf{W}=[F_x, F_y, F_z, M_x, M_y, M_z]^T$ contains the forces (F_x, F_y, F_z) and moments (M_x, M_y, M_z) acting on the robot's center of gravity and represents the robot's payload, including the effect of surface gradient, any externally applied forces and inertial effects of the robot's body (refer to Figure 4). However, the inertial effects of the legs have been neglected to simplify the study. The trunk body is held at a constant height and parallel to the ground plane during locomotion. Under these conditions, six equilibrium equations that balance forces and moments can be written in matrix form as given below.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & -z_1 & y_1 & 0 & -z_2 & y_2 & 0 & -z_3 & y_3 \\ z_1 & 0 & -x_1 & z_2 & 0 & -x_2 & z_3 & 0 & -x_3 \\ -y_1 & x_1 & 0 & -y_2 & x_2 & 0 & -y_3 & x_3 & 0 \end{bmatrix} \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{z1} \\ f_{x2} \\ f_{y2} \\ f_{z2} \\ f_{x3} \\ f_{y3} \\ f_{z3} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{11}$$

The above matrix can be written as follows:

$$[\mathbf{C}].[\mathbf{F}] = [\mathbf{W}] \tag{12}$$

The coordinates of i^{th} foot-ground contact point with respect to body reference frame, located at the body's geometric center, are denoted by (x_i, y_i, z_i).

With the known feet positions, the feet forces during a whole locomotion cycle can be computed using equation (12), which is indeterminate, because it consists of six equations and nine unknowns. The solution of equation (12) has been obtained using the least squared method [18], which gives the minimum norm solution of the indeterminate equilibrium equations. In other words, it is the solution that minimizes the sum of the squares of components of feet forces. The solution is written in a matrix form as given below.

$$[\mathbf{F}] = [\mathbf{C}]^{\mathbf{T}} \left[\mathbf{C}.\mathbf{C}^{\mathbf{T}}\right]^{-1} [\mathbf{W}]$$

Approach 2: Minimization of Norm of Joint Torques

In this approach, the equation (12) can be re-formulated by using the following relations.

$$[\mathbf{F}] = [\mathbf{D}].[\boldsymbol{\tau}] \tag{13}$$

where $[\mathbf{D}] = \begin{bmatrix} {}^{1}\mathbf{J} & \mathbf{0_3} & \mathbf{0_3} \\ \mathbf{0_3} & {}^{2}\mathbf{J} & \mathbf{0_3} \\ \mathbf{0_3} & \mathbf{0_3} & {}^{3}\mathbf{J} \end{bmatrix}$; ${}^{i}\mathbf{J} = \left[\mathbf{J}_i^T\right]^{-1}$; and $\mathbf{J}_i$ is the (3×3) Jacobian matrix of leg i and $\mathbf{0}_3$ is the (3×3) null matrix. Here, $[\boldsymbol{\tau}]=[\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \boldsymbol{\tau}_3]^T$ and $\boldsymbol{\tau}_i=[\tau_{1i}, \tau_{2i}, \tau_{3i}]^T$ is the torque vector containing three joint torques at leg i (i=1, 2, 3).

The equation (12) can be re-written as follows:

$$[\mathbf{C}].[\mathbf{D}].[\boldsymbol{\tau}] = [\mathbf{W}] \tag{14}$$

$$[\mathbf{C_D}].[\boldsymbol{\tau}] = [\mathbf{W}] \tag{15}$$

The minimum norm solution of the above indeterminate equations (i.e., solution that minimizes the sum of the squares of joint torques) has been obtained using a least squared method. The solution is written in a matrix form as given below.

$$[\boldsymbol{\tau}] = [\mathbf{C_D}]^{\mathbf{T}} \left[\mathbf{C_D}.\mathbf{C_D^T}\right]^{-1} [\mathbf{W}]$$

Feet forces can be determined with the help of equation (13).

Approach 3: Minimization of Norm of Joint Power

In this approach, the equation (13) can be re-formulated by using the following relations.

$$[\boldsymbol{\tau}] = [\mathbf{V}].[\mathbf{P}] \tag{16}$$

where $[\mathbf{V}] = \begin{bmatrix} {}^{1}\boldsymbol{\omega} & \mathbf{0_3} & \mathbf{0_3} \\ \mathbf{0_3} & {}^{2}\boldsymbol{\omega} & \mathbf{0_3} \\ \mathbf{0_3} & \mathbf{0_3} & {}^{3}\boldsymbol{\omega} \end{bmatrix}$; ${}^{i}\boldsymbol{\omega} = \begin{bmatrix} \dot{\theta}_{1i} & 0 & 0 \\ 0 & \dot{\theta}_{2i} & 0 \\ 0 & 0 & \dot{\theta}_{3i} \end{bmatrix}^{-1}$ and $\left(\dot{\theta}_{1i} \quad \dot{\theta}_{2i} \quad \dot{\theta}_{3i}\right)$ are three joint velocities of leg i. Here, $[\mathbf{P}]=[\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3]^T$ and $\mathbf{P}_i=[p_{1i}, p_{2i}, p_{3i}]^T$ is the joint power matrix containing three joint mechanical power at leg i (i=1, 2, 3).

The equation (13) can be re-written as follows:

$$[\mathbf{F}] = [\mathbf{D}].[\mathbf{V}].[\mathbf{P}] \tag{17}$$

$$[\mathbf{C}].[\mathbf{D}].[\mathbf{V}].[\mathbf{P}] = [\mathbf{W}] \tag{18}$$

$$[\mathbf{C_{DV}}].[\mathbf{P}] = [\mathbf{W}] \tag{19}$$

The minimum norm solution of the above indeterminate equations (i.e., solution that minimizes the sum of the squares of joint mechanical power) has been obtained using a least squared method. The solution is written in a matrix form as given below.

$$[\mathbf{P}] = [\mathbf{C_{DV}}]^{\mathbf{T}} \left[\mathbf{C_{DV}}.\mathbf{C_{DV}^T}\right]^{-1} [\mathbf{W}]$$

Feet forces can be obtained with the help of equations (13) and (16).

Once the torques required at various joints are calculated using equation (10), the amount of power consumed at those joints can be estimated. At a given joint 'i', the required mechanical power is calculated as:

$$P_i = \frac{1}{T}\int_0^T \left|\tau_i \theta_i\right| dt \tag{20}$$

Therefore, the total power consumed by all joint (number of joints=12) can be determined as follows:

$$P=\sum_{i=1}^{12} P_i \tag{21}$$

3. RESULTS AND ANALYSIS

In this section, simulation results of the proposed three approaches have been discussed in detail. Table 2 shows the physical parameters of each leg of a real four-legged robot (computed utilizing CATIA solid modeling software package), which have been used in simulations. In computer simulations, the walking parameters, like height of the trunk body, velocity of the body, stroke and duty factor, are fed as inputs, whereas the distributions of feet forces and joint torques are considered as the outputs. The cycle time, leg stroke, body height, and velocity of the trunk body are assumed to be equal to 4 sec, 0.15 m, 0.13 m and 0.05 m/sec, respectively.

Table 2. Physical parameters of each leg of the four-legged robot

Link parameters		Link 1	Link 2	Link 3
Mass (kg)	m	0.152	0.04	0.106
Length (m)	L	0.085	0.115	0.100
Position of Center of	x	-71.22	-71.40	-97.33
mass (10^{-3} m)	y	-14.04	-2.47	0.98
	z	0.00	8.21	-3.43
Moment of Inertia	I_{xx}	1.00	0.23	0.22
(10^{-4} kg-m^2)	I_{yy}	8.28	3.07	10.00
	I_{zz}	9.09	2.91	10.01
Product of Inertia	I_{xy}	-1.57	-0.141	0.103
(10^{-4} kg-m^2)	I_{xz}	-0.113	0.364	-0.376
	I_{yz}	-0.037	0.018	0.0036

Figures 5, 6 and 7 show the distributions of foot forces obtained by approaches 1, 2, and 3, respectively over one locomotion cycle. It shows that the front and rear legs complement each other in force, such that the sum of vertical forces of all the ground legs at any given instant of time becomes equal to the weight of the robot. Approach 1 has yielded the forces with either zero or almost zero horizontal components during the phase of constant velocity of the trunk body; therefore, the robot does not make a good use of the friction. However, in approaches 2 and 3, horizontal components of the foot forces are found to be significant. These results are quite similar to that reported by Erden and Leblebicioglu [15].

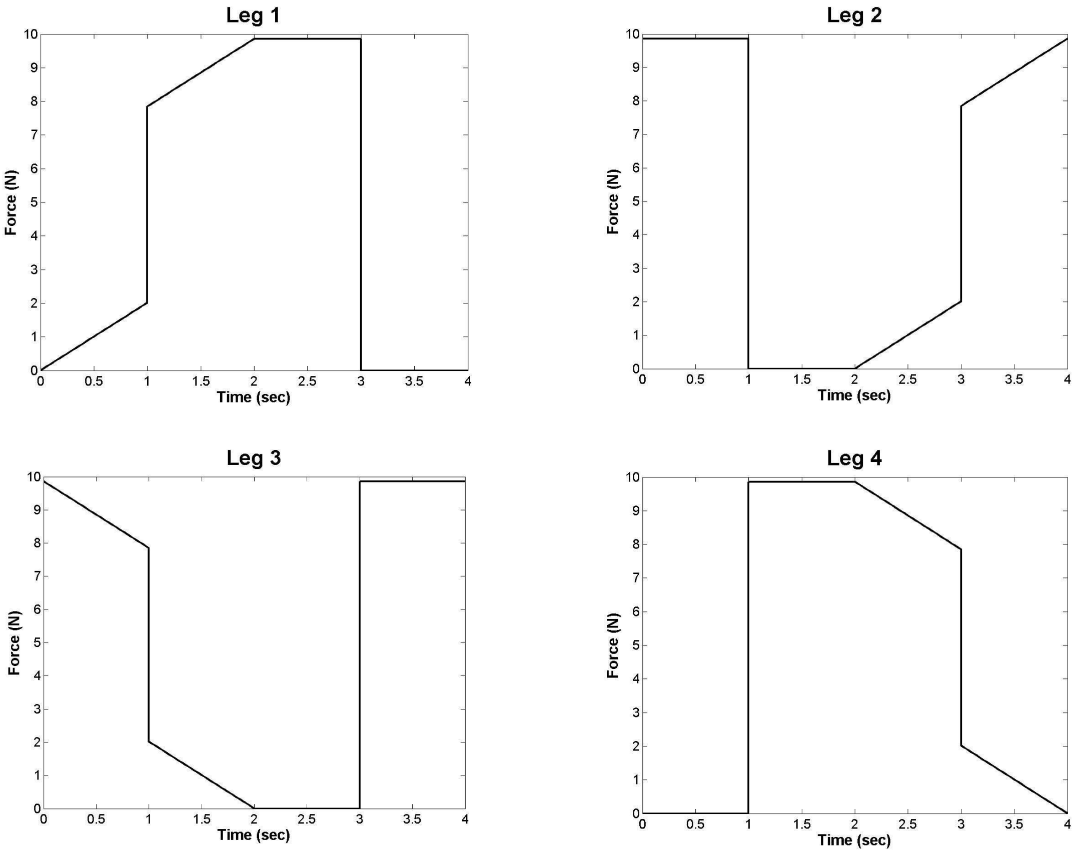

Figure 5. Distributions of feet forces obtained by approach 1

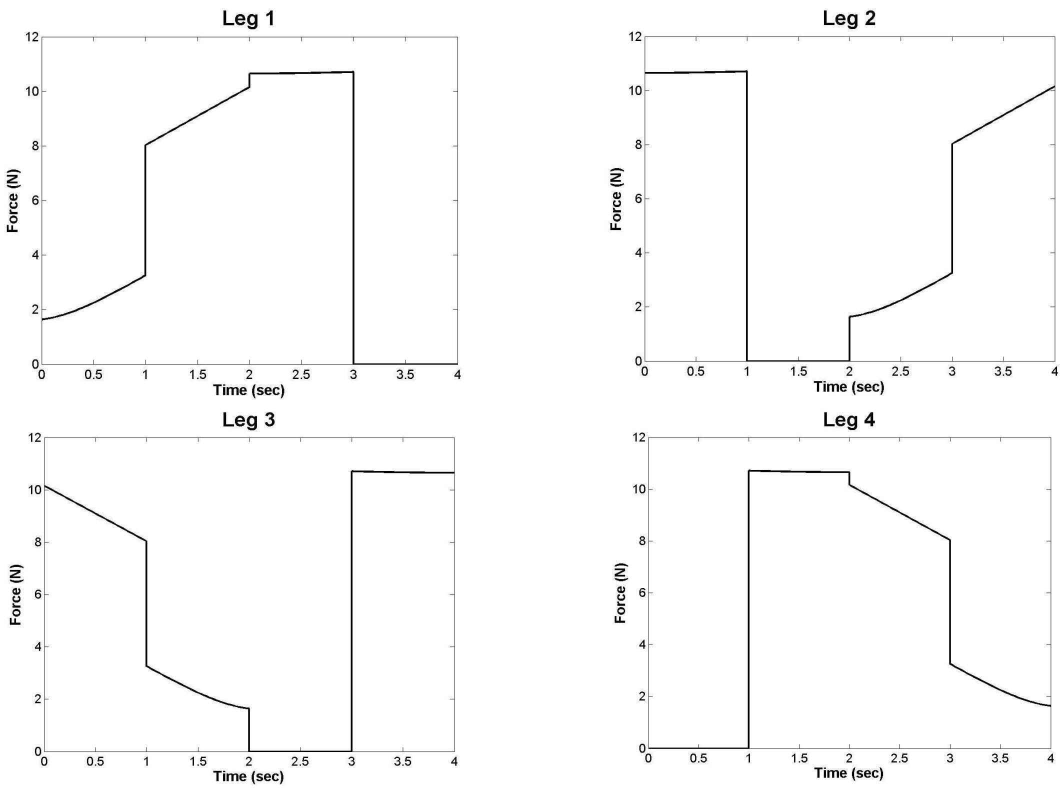

Figure 6. Distributions of feet forces obtained by approach 2

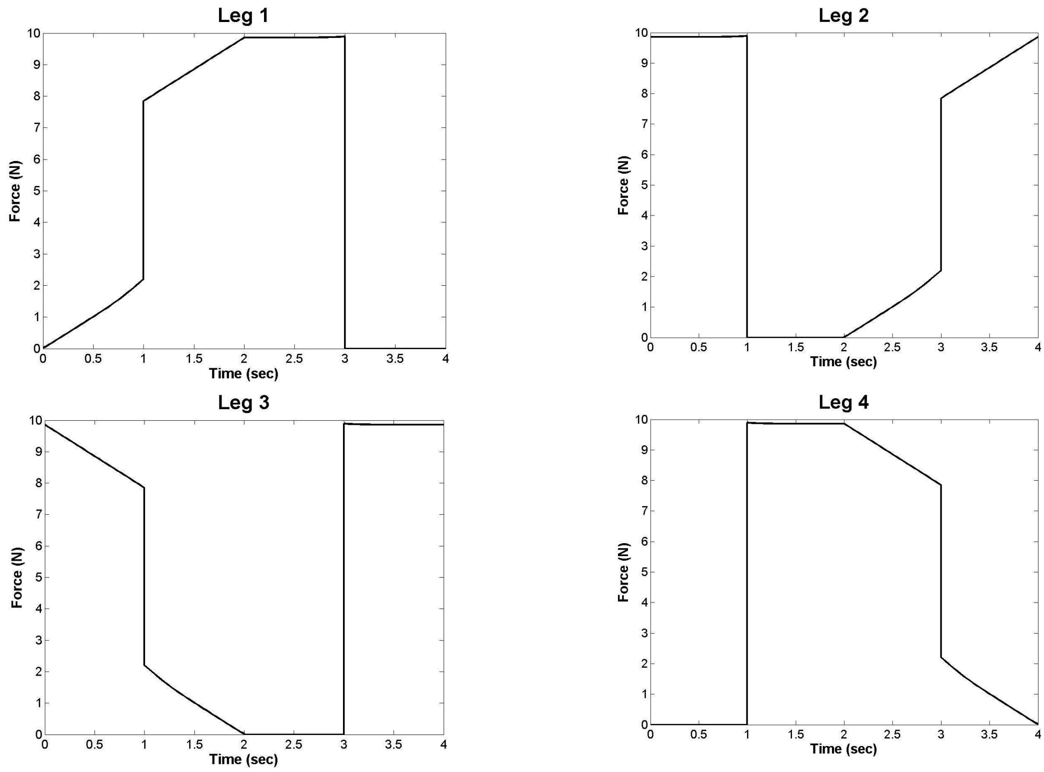

Figure 7. Distributions of feet forces obtained by approach 3

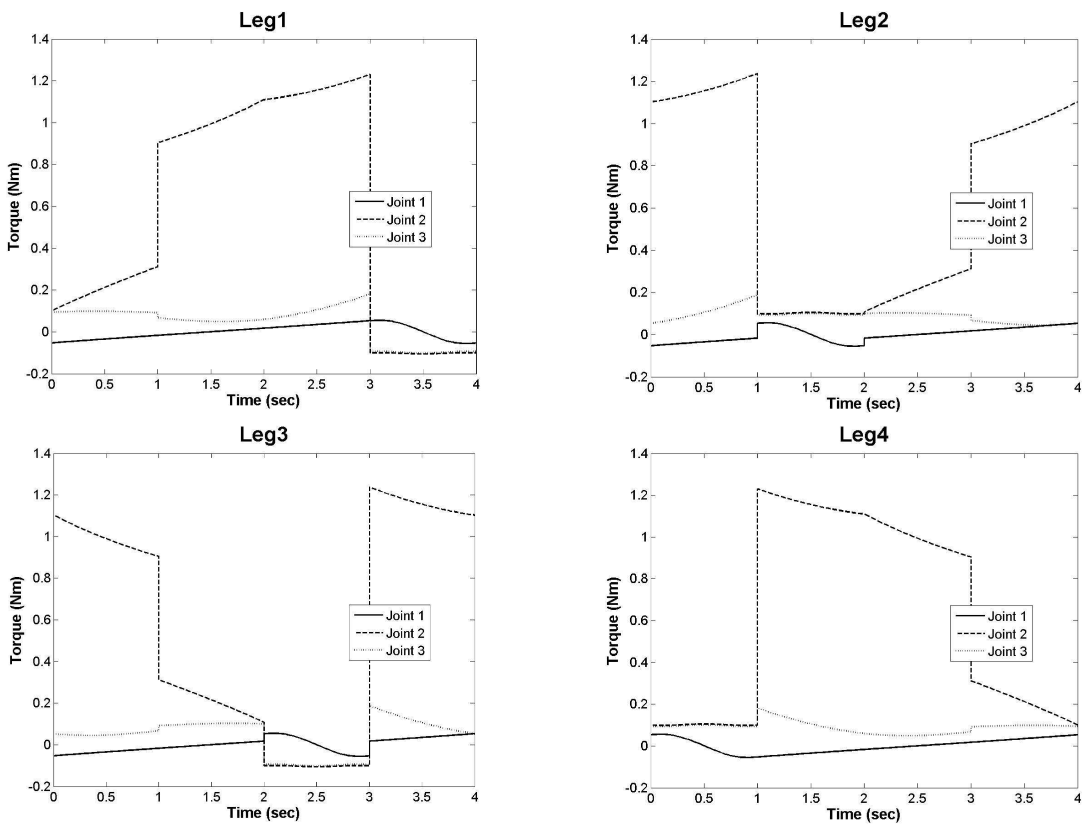

Figure 8. Variations of joint torques at each joint of the legs obtained by Approach 1

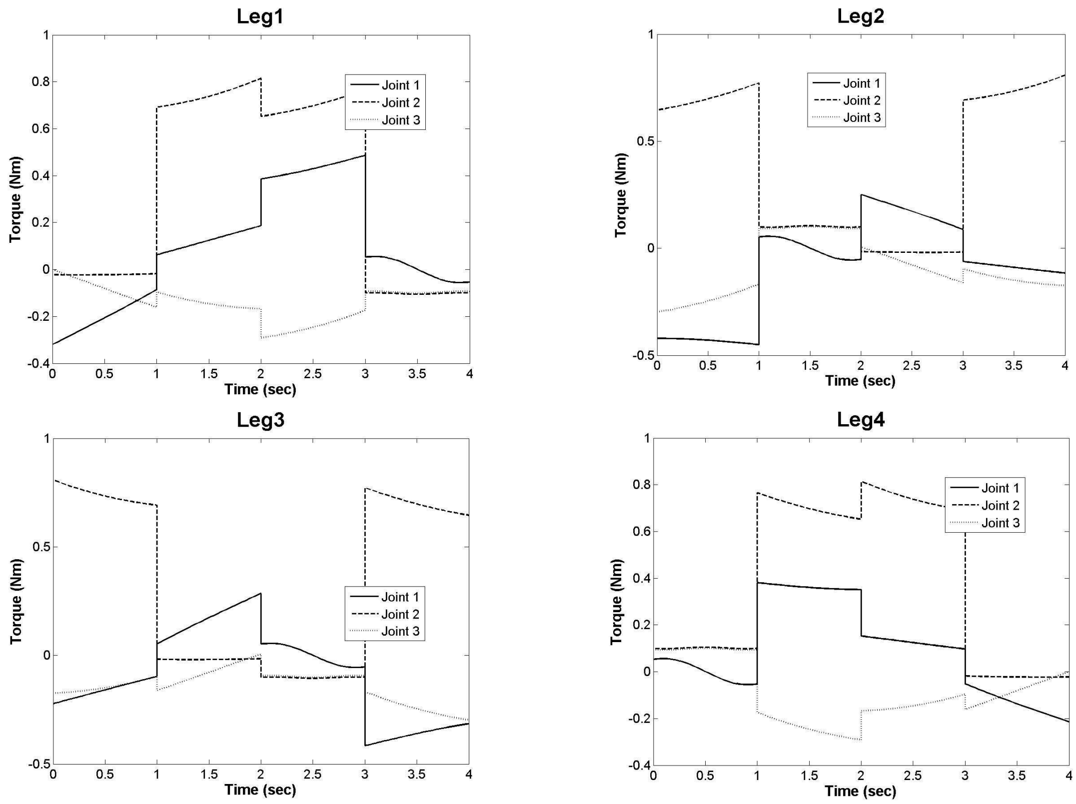

Figure 9. Variations of joint torques at each joint of the legs obtained by Approach 2

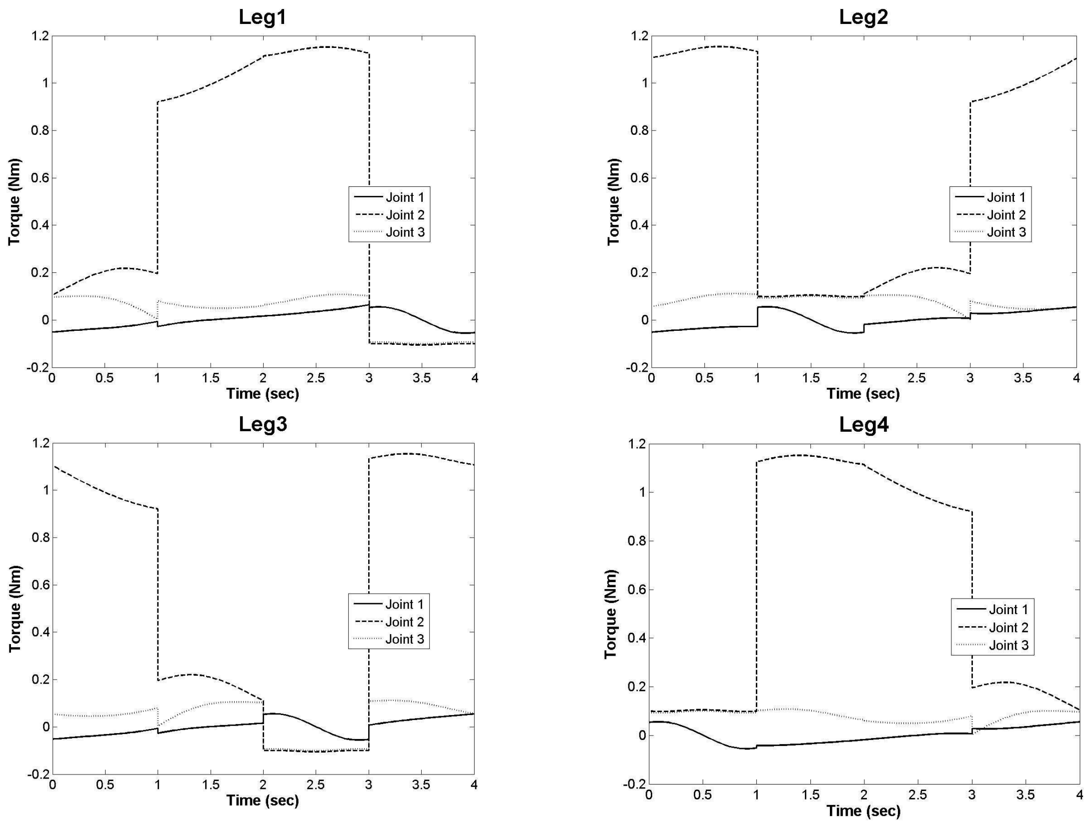

Figure 10. Variations of joint torques at each joint of the legs obtained by Approach 3

Figures 8 to 10 represent the variations of torques at each joint of the legs during one locomotion cycles as obtained by approaches 1 to 3. It is interesting to note that for a particular ground leg, the maximum torque generated at joint 2 has turned out to be more compared to that at other two joints. Moreover, joint torques of the legs during the support phase have been found to be more than those during the transfer phase, as expected. A close watch on Figure 8 (that is, result of Approach 1) indicates that joint 2 of each leg is subjected to the maximum amount of torque than that at other joints (namely joints 1 and 3). Therefore, if we select joint motors based on the torque requirement at joint 2, these motors will be under-utilized at joints 1 and 3. Otherwise, the size and capacity variations of different joint motors will be significant. In approach 2, the variations of torque requirement at different joints of the middle and other legs are seen to be relatively less than that of approach 1. Thus, in approach 2, the variations among joint motors will be less compared to that in approach 1.

Table 3. Average values of the power consumption obtained by three approahes

Approaches	Average power consumption (Watts)
Approach 1	0.0731
Approach 2	0.1551
Approach 3	0.0706

Table 3 shows the average values of the joint power of the four-legged robot obtained by three approaches. The average value of joint power of the robot as obtained by approach 3 is seen to be lesser than that yielded by other two approaches. It can be concluded that by approach 3 is more energy efficient foot force formulation than other two approaches.

4. CONCLUSION

In the present work, both the kinematics and dynamics of a four-legged robots' locomotion has been solved. An attempt has also been made in present study to obtain optimal distributions of feet forces. Three approaches, namely, minimization of norm of feet forces (approach 1), minimization of norm of joint torques (approach 2) and minimization of norm of joint work (approach 3) have been developed. It is important to mention that approach 3 is seen to be more energy efficient compared to other two approaches. Joint torques have been estimated using Lagrange-Euler formulation of the rigid multi-body system. The developed kinematic and dynamic models have been examined for continuous gait of the said robot. This work can be extended to tackle the problems related to discontinuous and non-periodic gait of the multi-legged walking robots.

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