Information Retrieval
CS 6900

Lecture 03

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Statistical Properties of Text

- **Zipf's Law** models the distribution of terms in a corpus:
  - How many times does the k^{th} most frequent word appears in a corpus of size N words?
  - Important for determining index terms and properties of compression algorithms.

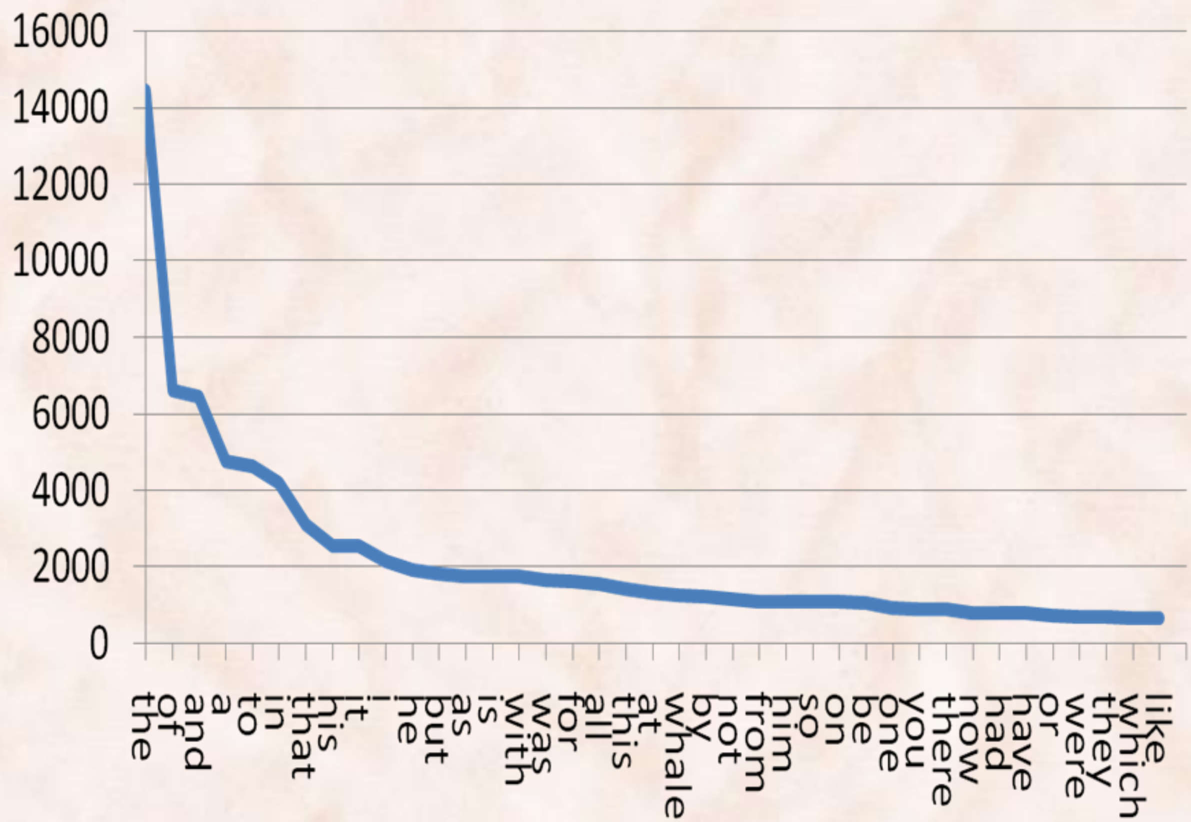
- **Heap's Law** models the number of words in the vocabulary as a function of the corpus size:
  - What is the number of unique words appearing in a corpus of size N words?
  - This determines how the size of the inverted index will scale with the size of the corpus .

Word Distribution

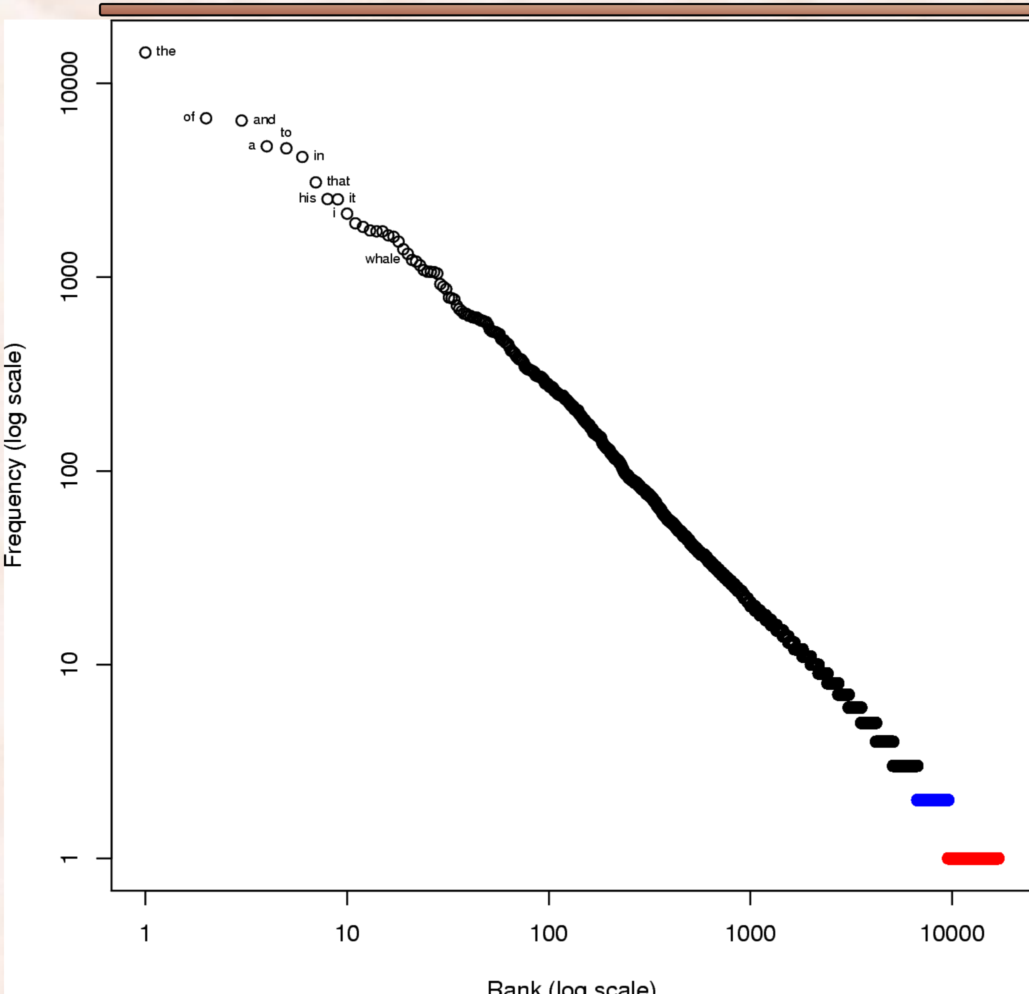
- **A few words are very common**:
  - The 2 most frequent words (e.g. “the”, “of”) can account for about 10% of word occurrences.
- **Most words are very rare**:
  - Half the words in a corpus appear only once, called *hapax legomena* (Greek for “read only once”)
- A “*heavy tailed*” or “*long tailed*” distribution:
  - Since most of the probability mass is in the “tail” compared to an exponential distribution.

Word Distribution

Frequency vs. rank for all words in Moby Dick



Word Distribution (Log Scale)



Moby Dick:
- 44% *hapax legomena*
- 17% *dis legomena*

“Honorificabilitudinitatibus”:
- Shakespeare’s *hapax legomenon*
- longest word with alternating vowels and consonants

Zipf's Law

- Rank all the words in the vocabulary by their frequency, in decreasing order.
  - Let $r(w)$ be the rank of word w.
  - Let $f(w)$ be the frequency of word w.

- Zipf (1949) postulated that frequency and rank are related by a *power law*:

$$f(w) = \frac{c}{r(w)}$$

  - c is a normalization constant that depends on the corpus.

Zipf's Law

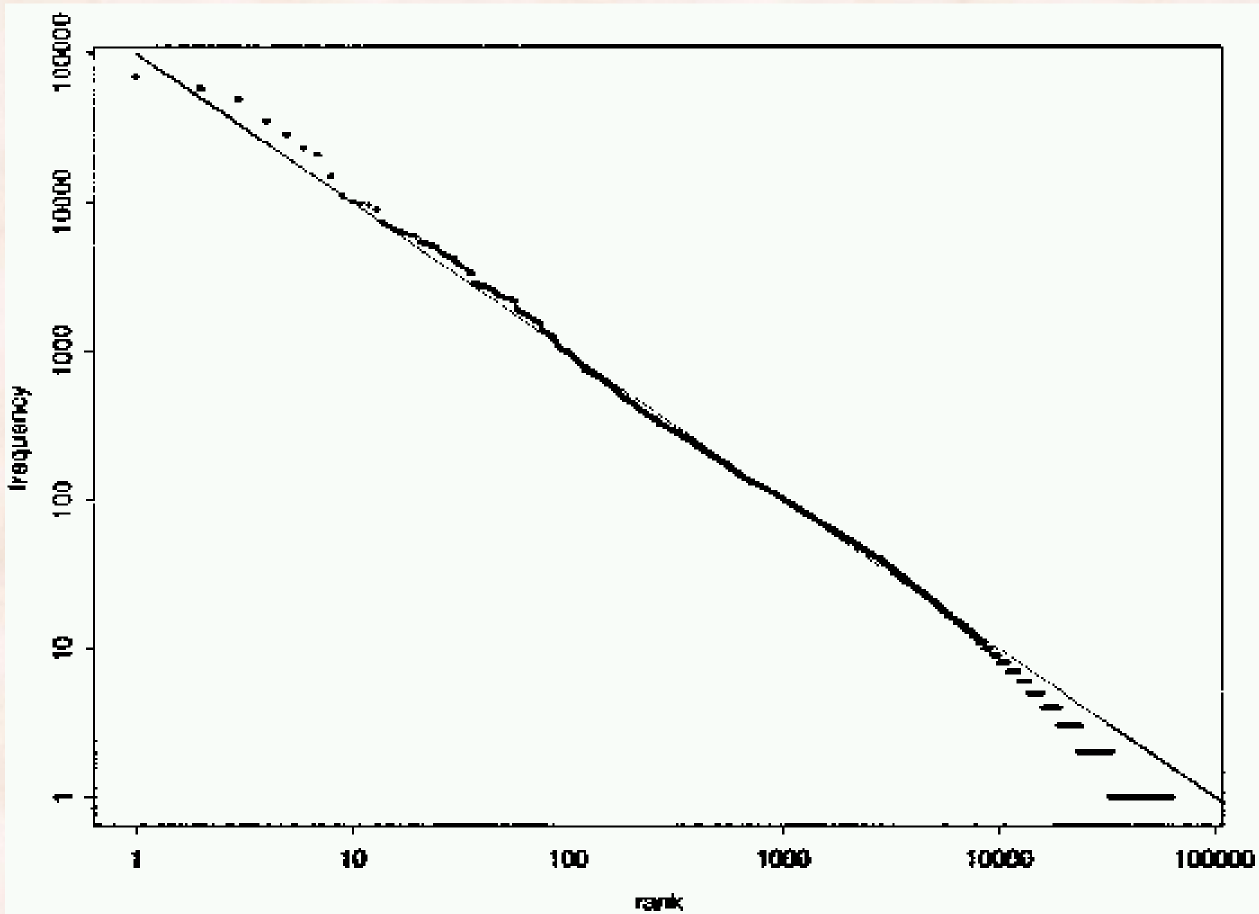
- If the most frequent term (the) occurs f_1 times:
  - Then the second most frequent term (of) occurs $f_1 / 2$ times.
  - The third most frequent term (and) occurs $f_1 / 3$ times, …

- **Power Laws**: $y = cx^k$
  - Zipf's Law is a power law with $k = -1$.
  - Linear relationship between $\log(y)$ and $\log(x)$:
    - $\log(y) = \log c + k \log(x)$
    - on a log scale, power laws give a straight line with slope k.

- Zipf is quite accurate, except for very high and low rank.

Zipf's Law Fit to Brown Corpus

$$f(w) = \frac{100000}{r(w)}$$



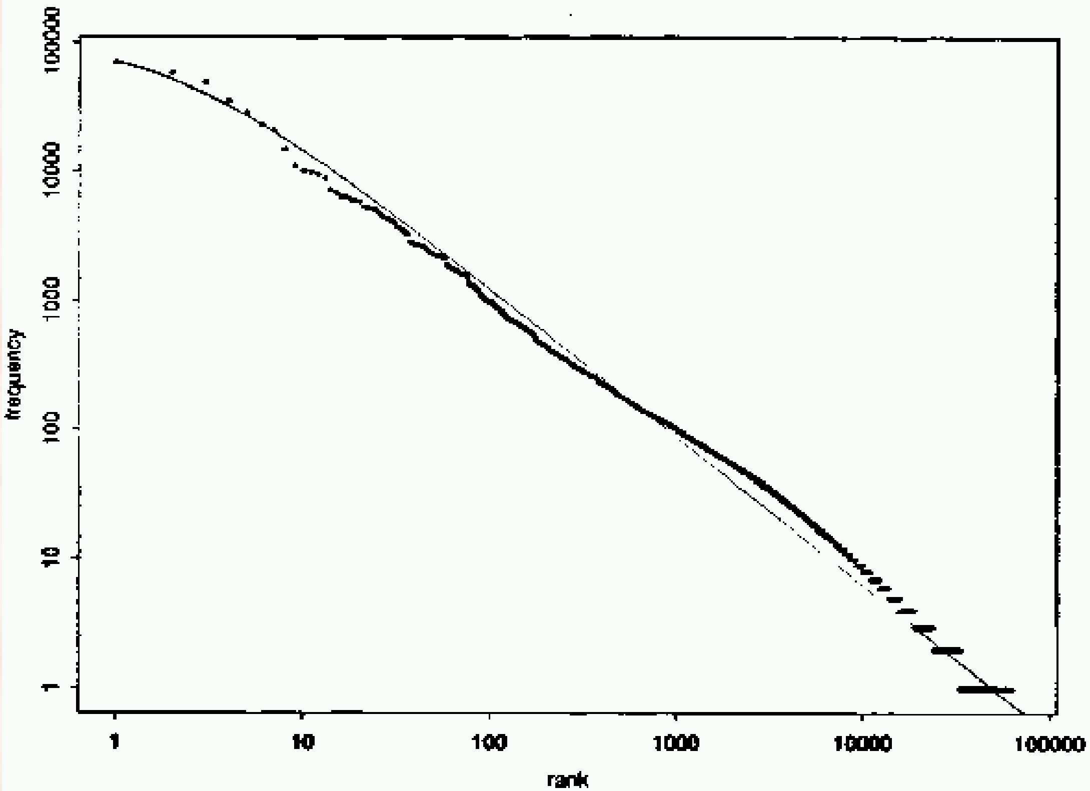
Mandelbrot's Distribution

- The following more general form gives a bit better fit:

$$f = c/(r+\rho)^K$$

- When fit to Brown corpus:
  - $c = 105.4$
  - $K = -1.15$
  - $\rho = 100$

Mandelbrot's Law Fit to Brown Corpus



Mandelbrot's function on Brown corpus

Zipf's Law Impact on IR

- Good News:
  - Stopwords will account for a large fraction of text, so eliminating them greatly reduces inverted-index storage costs.
  - Postings list for most remaining words in the inverted index will be short since they are rare, making retrieval fast.

- Bad News:
  - For most words, gathering sufficient data for meaningful statistical analysis is difficult since they are extremely rare.
    - for correlation analysis for query expansion.
    - for ML estimation in language modeling.

Vocabulary vs. Collection Size

- How big is the term vocabulary?
  - That is, how many distinct words are there?
- Can we assume an upper bound?
  - Not really upper-bounded due to proper names, typos, etc.
- In practice, the vocabulary will keep growing with the collection size.

Heap's Law

- Given:
  - M is the size of the vocabulary.
  - T is the number of tokens in the collection.
- Then:
  - $M = kT^b$
  - k, b depend on the collection type:
    - typical values: $30 \leq k \leq 100$ and $b \approx 0.5$ (square root).
    - in a log-log plot of M vs. T, Heaps' law predicts a line with slope of about ½.

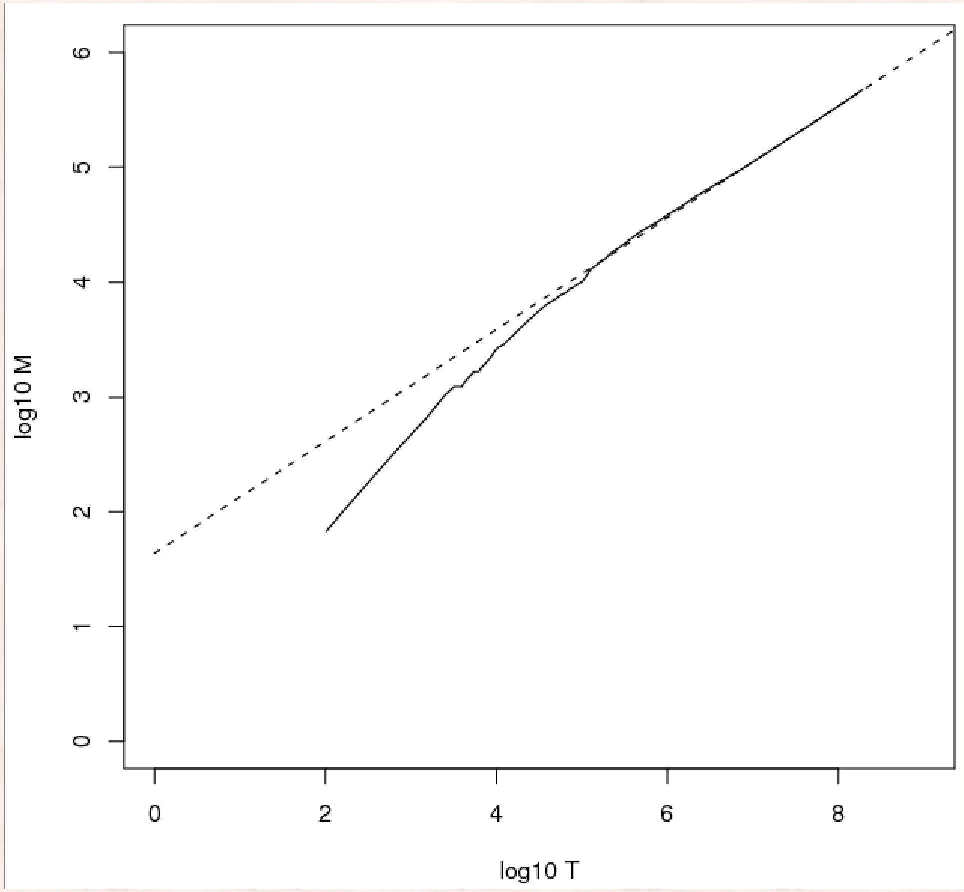
Heap's Law Fit to Reuters RCV1

- For RCV1, the dashed line
  $\log_{10} M = 0.49 \log_{10} T + 1.64$
  is the best least squares fit.

- Thus, $M = 10^{1.64} T^{0.49}$ so
  $k = 10^{1.64} \approx 44$ and $b = 0.49$.

- For first 1,000,020 tokens:
  - Law predicts 38,323 terms;
  - Actually, 38,365 terms.

  ⇒ Good empirical fit for RCV1!



Explanations

- **Zipf's Law**:
  - Zipf's explanation was his "principle of least effort":
    - Balance between speaker's desire for a small vocabulary and hearer's desire for a large one.
  - Herbert Simon's explanation is "rich get richer."
  - Li (1992) shows that just random typing of letters including a space will generate "words" with a Zipfian distribution.

- **Heaps' Law**:
  - Can be derived from Zipf's law by assuming documents are generated by randomly sampling words from a Zipfian distribution.