HW Assignment 2 (Due date: February 13, by 9:00 am)

1. [**Sorting Complexity, 5 points**] Exercise 1.2-2, page 14.
2. [**Time Complexity, 10 points**] Exercise 2.2-3, page 29.
3. [**Binary Search, 5 points**] Exercise 2.3-5, page 39.
4. [**Binary Search, 5 points**] Exercise 2.3-6, page 39.
5. [**Asymptotic Notation, 5 points**] Exercise 3.1-1, page 52.
6. [**Asymptotic Notation, 10 points**] Prove or disprove:
   a) $3^{n+1} = O(3^n)$
   b) $2^{2n} = O(2^n)$
   c) $3n^2 \lg n + 4n = O(n^3)$
   d) $3n^2 \lg n + 4n = O(n^2 \lg n)$
   e) $3n^2 \lg n + 4n = O(n^2 \sqrt{n})$
7. [**Asymptotic Notation, 5 points**] Prove $\lg(n!) = \theta(n \lg n)$.
   [Hint: use one of Stirling's approximations (3.18 or 3.20 on page 57)].
8. [**Substitution Method, 15 points**] Show that the solution to:
   $T(6) = 1$
   $T(n) = 3T(\lfloor n/3 \rfloor + 4) + n$, for $n > 6$
   is $O(n \lg n)$.
9. [**Master Method, 10 points**] Use the master method to give tight asymptotic bounds for the following recurrences:
   a) $T(n) = 4T(n/2) + n$.
   b) $T(n) = 4T(n/2) + n^2$.
   c) $T(n) = 4T(n/2) + n^3$.
   d) $T(n) = 2T(n/4) + \sqrt{n}$.
   e) $T(n) = 2T(n/4) + n^2$.
10. [**Master Method, 5 points**] The recurrence $T(n) = 10T(n/3) + n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/9) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?
11. [**Recurrence, 20 points**] Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \le 2$. Make your bounds as tight as possible, and justify your answers.

a) $T(n) = 2T(n/2) + n^3$.

b) $T(n) = T(9n/10) + n$.

c) $T(n) = 16T(n/4) + n^2$.

d) $T(n) = 7T(n/3) + n^2$.

e) $T(n) = 7T(n/2) + n^2$.

f) $T(n) = 2T(n/4) + \sqrt{n}$.

g) $T(n) = T(n-1) + n$.

h) $T(n) = T(\sqrt{n}) + 1$.

12. [**Recurrence, 5 points (*)**]. Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a change of variable. The solution should be asymptotically tight (i.e. use the Θ notation). Do not worry about whether values are integral.

13. [**Design & Analysis, 10 points (*)**] Exercise 2.3-7, page 39.