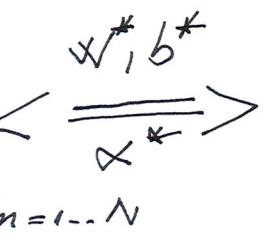


Separable case: Primal

$$\min_{w, b} \frac{1}{2} \|w\|^2$$

$$\text{subj. to } t_n (w^T x_n + b) \geq 1 \quad \forall n=1 \dots N$$



$$\alpha = [\alpha_1, \dots, \alpha_N]$$

$$\max_{\alpha} L_{\Delta}(\alpha)$$

$$\text{subj. to: } \alpha_n \geq 0, \quad \forall n=1 \dots N$$

$$L_{\Delta}(\alpha^*) = \frac{1}{2} \|w^*\|^2$$

$$L_{\Delta}(\alpha) = \inf_{w, b} \mathcal{L}(w, b, \alpha)$$

$$\mathcal{L}(w, b, \alpha) = \frac{1}{2} \|w\|^2 + \sum_{n=1}^N \alpha_n (1 - t_n (w^T x_n + b))$$

$$\frac{\partial \mathcal{L}}{\partial w} = 0 \Rightarrow w = \sum_{n=1}^N \alpha_n t_n x_n \quad (*)$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \Rightarrow \sum_{n=1}^N \alpha_n t_n = 0 \quad (**)$$

~~$$L_{\Delta}(\alpha) = \frac{1}{2} w^T w + \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \alpha_n t_n w^T x_n - b \sum_{n=1}^N \alpha_n t_n \quad (**)$$~~

$$= \sum_{n=1}^N \alpha_n + \frac{1}{2} w^T w - w^T \left(\sum_{n=1}^N \alpha_n t_n x_n \right)$$

$$= \sum_{n=1}^N \alpha_n + \frac{1}{2} w^T w - w^T w = \sum_{n=1}^N \alpha_n - \frac{1}{2} w^T w$$

$$L_{\Delta}(\alpha) = \sum_{n=1}^N \alpha_n - \frac{1}{2} \left(\sum_{m=1}^N \alpha_m t_m x_m^T \right) \left(\sum_{n=1}^N \alpha_n t_n x_n \right)$$

$$L_{\Delta}(x) = \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N \alpha_m \alpha_n t_m t_n \underbrace{x_m^T x_n}_{K(x_m, x_n)}$$

Dual SVM

$$\max_{\alpha} L_{\Delta}(x) = \sum_n \alpha_n - \frac{1}{2} \sum_m \sum_n \alpha_m \alpha_n t_m t_n K(x_m, x_n)$$

$$\text{subj. to } \alpha_n \geq 0, \forall n=1..N$$

$$\sum_{n=1}^N \alpha_n t_n = 0$$

KKT Conditions:

$$t_n (w^T x_n + b) \geq 1$$

$$\alpha_n \geq 0$$

$$\alpha_n \cdot (1 - t_n (w^T x_n + b)) = 0$$

$$\forall n=1..N$$

$$w = \sum_{n=1}^N \alpha_n t_n x_n$$

$$w = \sum_{m \in S} \alpha_m t_m x_m$$

$$S = \{n \mid \alpha_n > 0\}$$

either $\alpha_n = 0$ or ($\alpha_n > 0$ and $t_n (w^T x_n + b) = 1$)

$$w^T x_n + b = t_n \Rightarrow b = t_n - w^T x_n, \forall n=1..N$$

$$b = \frac{1}{N} \sum_{n=1}^N \left(t_n - \sum_{m \in S} \alpha_m t_m x_m^T x_n \right)$$

Test time: $w^T x + b \geq 0?$

$$\sum_{m \in S} \alpha_m t_m \underbrace{x_m^T x}_{K(x_m, x)} +$$