$$J(w) = \frac{1}{2N}\sum_{n=1}^{N}\left(w^T x^{(n)} - t_n\right)^2 \qquad \hat{w} = \underset{w}{\arg\min}\, J(w)$$

$\hookrightarrow [w_0, w_1, \dots, w_M]^T$

$$\frac{\partial J}{\partial w} = 0 \quad \underline{\text{Vectorized form}}: \ \frac{1}{2N}\sum_{n=1}^{N} 2\left(w^T x^{(n)} - t_n\right) \cdot \underbrace{\frac{\partial w^T x^{(n)}}{\partial w}}_{x^{(n)}} = 0$$

$$\Rightarrow \frac{1}{N}\sum_{n=1}^{N}\left(w^T x^{(n)} - t_n\right) x^{(n)} = 0$$

$$\frac{\partial w^T x^{(n)}}{\partial w} = \left[\frac{\partial w^T x^{(n)}}{\partial w_0}, \frac{\partial w^T x^{(n)}}{\partial w_1}, \dots, \frac{\partial w^T x^{(n)}}{\partial w_M}\right]$$

$$x^{(n)} = [x_0^{(n)}, x_1^{(n)}, \dots, x_M^{(n)}]$$

$x_0 \hookrightarrow 1$

$$= \left[x_0^{(n)}, x_1^{(n)}, \dots, x_M^{(n)}\right] = x^{(n)}$$

$$\underline{w^T x^{(n)}} = w_0 x_0^{(n)} + w_1 x_1^{(n)} + \dots + w_M^{(n)} x_M^{(n)}$$

$$\text{matrix } X \xrightarrow{\text{MP pseudo-inverse}} (X^T X)^{-1} X^T \Rightarrow n \times m$$

$X \hookrightarrow m \times n$

$(X^T X) \hookrightarrow n \times n$, $X^T \hookrightarrow n \times m$

$$(X^T X)^{-1}(X^T X) = I$$