lr-numpy

April 7, 2021

1 Logistic Regression in NumPy

In this assignment, you will implement the logistic regression model using NumPy and evaluate it on 3 artificial datasets: *shapes*, *flower*, and *spiral*.

1.1 Write Your Name Here:

2 Submission Instructions

1. Click the Save button at the top of the Jupyter Notebook.
2. Please make sure to have entered your name above.
3. Select Cell -> All Output -> Clear. This will clear all the outputs from all cells (but will keep the content of ll cells).
4. Select Cell -> Run All. This will run all the cells in order, and will take several minutes.
5. Once you've rerun everything, select File -> Download as -> PDF via LaTeX and download a PDF version *lr-numpy.pdf* showing the code and the output of all cells, and save it in the same folder that contains the notebook file *lr-numpy.ipynb*.
6. Look at the PDF file and make sure all your solutions are there, displayed correctly. The PDF is the only thing we will see when grading!
7. Submit **both** your PDF and notebook on Canvas.

[5]:
```
import numpy as np
from scipy.sparse import coo_matrix
from numpy.random import randn, randint
from numpy.linalg import norm
import matplotlib.pyplot as plt

import utils

np.random.seed(1)
```

2.1 1. The Logistic Regression model

In this part, you will write code to:

1. Compute the cost and the gradient for the logistic regression model.

2. Compute the model predictions.

While not essential for this assignment, it is important to vectorize your code so that it runs quickly on larger datasets.

Implement the *softmax()* function, which computes and return the cost and its gradient with respect to the parameters **W** and **b**.

- **Cost & Gradient**: The cost and gradient should be computed according to the formulas shown on the slides, modified however to represent explicitly the bias and its gradient. Thus, if there are D features and K classes, the softmax model will be comprised of two types of parameters: **W** will be a K x D matrix of the feature weights, whereas **b** will be a K x 1 vector of bias terms.
- **Ground truth**: The *groundTruth* is a matrix M such that M[c, n] = 1 if sample n has label c, and 0 otherwise. This can be done quickly, without a loop, using the SciPy function *sparse.coo_matrix()*. Specifically, coo_matrix((data, (i, j))) constructs a matrix A such that A[i[k], j[k]] = data[k], where the shape is inferred from the index arrays. Sample code for computing the ground truth matrix has been provided in the lecture slides.
- **Overflow**: Make sure that you prevent overflow when computing the softmax probabilities, as shown on the slides.

[1]:
```
def softmaxCost(W, b, numClasses, inputSize, decay, data, labels):
    """Computes and returns the (cost, gradient)
    Args:
        W, b - the weight matrix and bias vector parameters.
        numClasses - the number of classes.
        inputSize - the size D of each input vector.
        decay - weight decay parameter.
        data - the D x N input matrix, where each column data[:,n] corresponds␣
↪to a single sample.
        labels - an N x 1 matrix containing the labels corresponding for the␣
↪input data.

    Returns:
        The cost, W's gradient, and b's gradient.
    """

    N = data.shape[1]

    groundTruth = coo_matrix((np.ones(N, dtype = np.uint8), (labels, np.
↪arange(N)))).toarray()
    cost = 0;
    dW = np.zeros((numClasses, inputSize))
    db = np.zeros((numClasses, 1))

    ## ---------- YOUR CODE HERE --------------------------------------
    #  Instructions: Compute the cost and gradient for softmax regression.
    #                You need to compute dW, dW, and cost.
    #                The groundTruth matrix might come in handy.
    return cost, dW, db
```

Implement the *softmaxPredict()* function, which computes the most likely label for each example in the input data matrix.

[3]:
```
def softmaxPredict(W, b, data):
    """Computes and returns the softmax predictions over the examples in data.
    Args:
        W, b - model parameters trained using softmaxTrain, a numClasses x D␣
↪matrix and a numClasses x 1 column vector.
        data - the D x N input matrix, where each column data[:,n] corresponds␣
↪to a single sample.

    Returns:
        pred - a vector of N predicted labels.
    """

    #  Your code should produce the prediction matrix pred,
    #  where pred(i) is argmax_c P(c | x(i)).

    ## ---------- YOUR CODE HERE ------------------------------------------
    #  Instructions: Compute pred using W and b, assuming that the labels
    #                start from 0.




    # --------------------------------------------------------------------

    return pred
```

2.2 2. Gradient Checking of the LR Model

In this part you will write code to compute the gradient numerically, and then compare the numerical gradient with the analytical gradient. They should be very close.

Implement the *computeNumericalGradient()* function

[4]:
```
def computeNumericalGradient(J, theta):
    """Compute numgrad = computeNumericalGradient(J, theta)
    Args:
        theta: A matrix of parameters (numClasses x D)
        J:     The cost function that outputs a real-number cost and the␣
↪gradient with respect to its parameters.
               Calling y = J(theta)[0] will return the cost function evaluated␣
↪at theta.

    Returns:
        numgrad: The matrix of partial derivatives with respect to the␣
↪parameters theta (numClasses x D)
    """

    # Initialize numgrad with zeros
    numgrad = np.zeros(theta.shape)

    ## ---------- YOUR CODE HERE --------------------------------------
    # Instructions:
    # Implement numerical gradient checking, and return the result in numgrad.
    # You should write code so that numgrad[i][j] is (the numerical␣
↪approximation to) the
    # partial derivative of J with respect to theta[i][j], evaluated at theta.
    # I.e., numgrad[i][j] should be the (approximately) partial derivative of J␣
↪with
    # respect to theta[i][j].
    #
    # Hint: You will probably want to compute the elements of numgrad one at a␣
↪time.









    ## ---------------------------------------------------------------

    return numgrad
```

Check that the numerically compute gradients are sufficiently close to the gradient computed analytically.

[ ]:
```
# Create a dataset with 100 examples, each with 8 random features.
inputSize = 8
instances, labels = randn(8, 100), randint(0, numClasses, 100, dtype = np.uint8)
```

```
# Randomly initialize parameters
W = 0.01 * randn(numClasses, inputSize)
b = np.zeros((numClasses, 1))

decay = 0.001
cost, dW, db = softmaxCost(W, b, numClasses, inputSize, decay, instances,␣
 ↪labels)
W_numGrad = computeNumericalGradient(lambda x: softmaxCost(x, b, numClasses,␣
 ↪inputSize,
                                                               decay, instances,␣
 ↪labels), W)

# Use this to visually compare the gradients side by side.
print(np.stack((W_numGrad.ravel(), dW.ravel())).T)

# Compare numerically computed gradients with those computed analytically.
diff = norm(W_numGrad - dW) / norm(W_numGrad + dW)
print(diff)

# The difference should be small.
# In our implementation, these values are usually less than 1e-7.
```

2.3 3. Experimental Evaluations

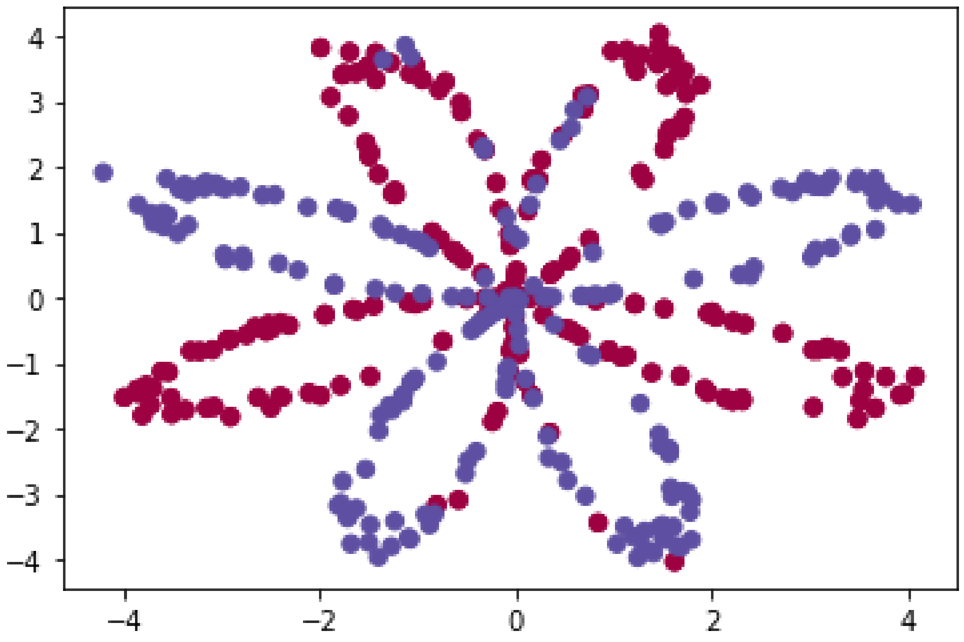
In this part, you will train and evaluate the LR model on 3 artificial datasets: *shapes*, *flower*, and *spiral*.

```
[ ]: def softmaxExercise(instances, labels, numClasses):
         """Train an evaluate the LR model.
         Args:
             theta: A matrix of parameters (numClasses x D)
             J:     The cost function that outputs a real-number cost and the␣
      ↪gradient with respect to its parameters.
                    Calling y = J(theta)[0] will return the cost function evaluated␣
      ↪at theta.

         Returns:
             numgrad: The matrix of partial derivatives with respect to the␣
      ↪parameters theta (numClasses x D)
         """

         # Randomly initialize parameters.
         W = 0.01 * randn(numClasses, inputSize)
         b = np.zeros((numClasses, 1))

         # Set hyper-parameters.
```

```
    learning_rate = 1.0
    decay = 0.001
    num_epochs = 200

    # Gradient descent loop.
    for epoch in range(num_epochs):
        # Call softmaxCost to compute cost and gradients.
        cost, dW, db = softmaxCost(W, b, numClasses, inputSize, decay,␣
 ↪instances, labels)
        if epoch % 10 == 0:
            print("Epoch %d: cost %f" % (epoch, cost))

        # Update parameters.
        W += -learning_rate * dW
        b += -learning_rate * db


    ## Testing on training data.
    #
    #  You should now test your model against the training examples.
    #  To do this, you will call softmaxPredict, which should return
    #  predictions given a softmax model and the input data.

    pred = softmaxPredict(W, b, instances)

    acc = np.mean(labels == pred)
    print('Accuracy: %0.3f%%.' % (acc * 100))

    # Accuracy is the proportion of correctly classified images
    # After 200 epochs, the results for our implementation were:
    #
    # Spiral Accuracy: 53.3%
    # Flower Accuracy: 47.0%

    return W, b
```

2.3.1 Evaluate the LR model on the *flower* dataset.

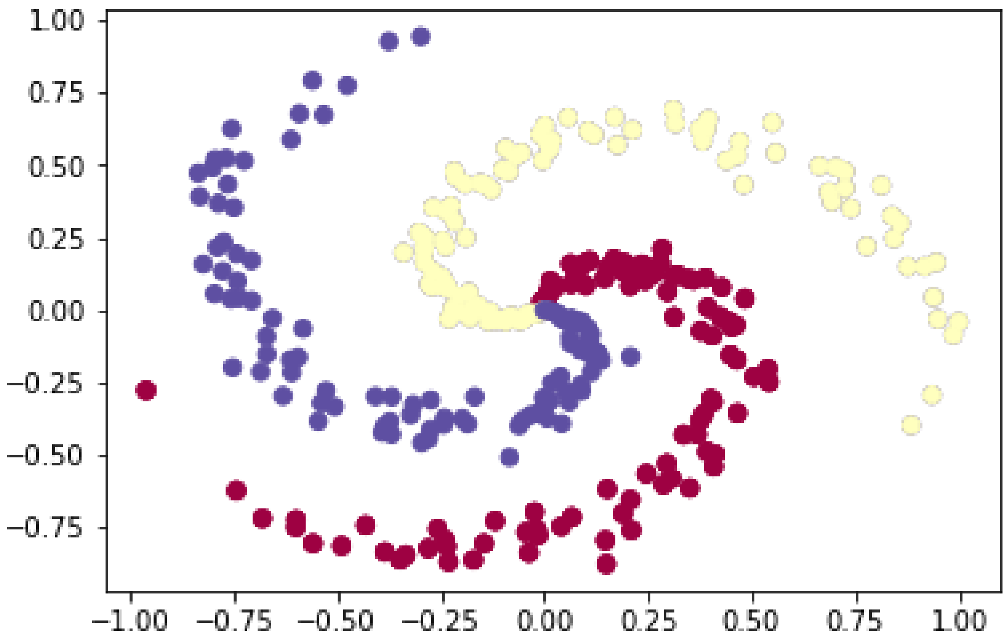
[7]:
```
instances, labels, numClasses = utils.load_flower_dataset()
W, b = softmaxExercise(instances, labels, numClasses)

# Plot the decision boundary.
utils.plot_decision_boundary(lambda x: softmaxPredict(W, b, x), instances,␣
 ↪labels)
plt.title("Softmax Regression")
plt.savefig('flower-boundary.jpg')
plt.show()
```



```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-7-635cab9ab7c5> in <module>
      1 instances, labels, numClasses = utils.load_flower_dataset()
----> 2 W, b = softmaxExercise(instances, labels, numClasses)
      3
      4 # Plot the decision boundary.
      5 utils.plot_decision_boundary(lambda x: softmaxPredict(W, b, x),␣
 ↪instances, labels)

NameError: name 'softmaxExercise' is not defined
```

2.3.2 Evaluate the LR model on the *spiral* dataset.

[8]:
```
instances, labels, numClasses = utils.load_spiral_dataset()
W, b = softmaxExercise(instances, labels, numClasses)

# Plot the decision boundary.
utils.plot_decision_boundary(lambda x: softmaxPredict(W, b, x), instances,␣
 ↪labels)
plt.title("Softmax Regression")
plt.savefig('spiral-boundary.jpg')
plt.show()
```



```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-8-54f7c75f5b74> in <module>
      1 instances, labels, numClasses = utils.load_spiral_dataset()
----> 2 W, b = softmaxExercise(instances, labels, numClasses)
      3 
      4 # Plot the decision boundary.
      5 utils.plot_decision_boundary(lambda x: softmaxPredict(W, b, x),␣
 ↪instances, labels)

NameError: name 'softmaxExercise' is not defined
```

2.3.3 Evaluate the LR model on the *shapes* dataset.

[ ]:
```
instances, labels, numClasses = # YOUR CODE HERE

W, b = softmaxExercise(instances, labels, numClasses)
```

[ ]:
```

```

2.4 4. Bonus points

- Use the *sklearn.linear_model.LogisticRegression* class from sklearn to run the 3 experimental evaluations above.

https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

- Anything extra goes here.

[ ]:

```
```

2.5 5. Analysis

Include here a nicely formatted report of the results, comparisons. Include explanations and any insights you can derive from the algorithm behavoir and the results. This section is important, so make sure you address it appropriately.

[ ]:

```
```