**Problem 1.** Find classification rules in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Car | Price | Mileage | Size | Accident | d |
| 1 | \* | \* | {full} | {doors, engine} | good |
| 2 | {low} | \* | {full} | {engine} | good |
| 3 | \* | {high} | {compact} | \* | poor |
| 4 | {high} | {low} | \* | {doors} | good |
| 5 | \* | \* | {full} | {doors} | excel |
| 6 | {low} | {high} | {compact} | \* | poor |

**Solution:**

Tolerance Classes New Decision Column D

T(1)={1,2, 4,5} D(1)={good, excel}

T(2)={2,1} D(2)={good}

T(3)={3,6} D(3)={poor}

T(4)={4,1,5} D(4)={good, excel}

T(5)={5.1,4} D(5)={excel, good}

T(6)={6,3} D(6)={poor}

Table with a new decision column D

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Car | Price | Mileage | Size | Accident | D |
| 1 | \* | \* | {full} | {doors, engine} | good, excel |
| 2 | {low} | \* | {full} | {engine} | good |
| 3 | \* | {high} | {compact} | \* | poor |
| 4 | {high} | {low} | \* | {doors} | good, excel |
| 5 | \* | \* | {full} | {doors} | good, excel |
| 6 | {low} | {high} | {compact} | \* | poor |

Discernibility Matrix for the new table with decision D

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** |
| **2** | Empty |  |  |  |  |
| **3** | S | S |  |  |  |
| **4** | - | P, A | M |  |  |
| **5** | - | A | S | - |  |
| **6** | S | S | - | P, M | S |

Looking for reducts (find discernibility function F):

F(P,M,S,A)= S\*S\*Empty\*(P+A)\*A\*M\*(P+M)= Empty /reduct – the whole set of attributes/

Looking for object reducts (Find discernibility function F for each object):

F(1)=Empty\*S = Empty , F(2)=Empty\*S(P+A)AS = Empty, F(3) = SM,

F(4)=(P+A)M(P+M)=(P+A)M= PM+AM

F(5)=AS, F(6)=S(P+M)= SP + SM

Rules:

For F(3) and F(6): (Size,Compact) ^ (Milage, High) -> poor sup=2

For F(4): (Price,high)^(Mileague, low) -> Good or Excel

For F(4): (Mileague,low)^(Accident,doors)-> Good or Excel

For F(5): (Size, full)^ (Accident,doors) -> Good or Excel

For F(6): (Size,low)^(Price, low)-> poor

**Problem 2 (Homework).** Find SVM classifier for the dataset below:

|  |  |  |  |
| --- | --- | --- | --- |
|  | a | b | c |
| x1 | 1 | 3 | 1 |
| x2 | 0 | 1 | 1 |
| x3 | 2 | 2 | 0 |
| x4 | 2 | 0 | 0 |

**Chart, box and whisker chart

Description automatically generated**

**Chart

Description automatically generated**

**Problem 3 (Homework).**

Find optimal reduct in Table T following RSH (RS Heuristic) strategy. Attribute f is the decision attribute.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | e | f |
| x1 | 1 | 1 | 0 | 0 | 2 | 1 |
| x2 | 0 | 0 | 0 | 1 | 2 | 0 |
| x3 | 1 | 1 | 0 | 1 | 0 | 0 |
| x4 | 1 | 0 | 0 | 2 | 0 | 0 |
| x5 | 0 | 2 | 1 | 2 | 1 | 0 |
| x6 | 2 | 2 | 1 | 0 | 0 | 1 |
| x7 | 2 | 0 | 1 | 0 | 1 | 1 |

Table T.

**Problem 4.**

Follow DEAR1, DEAR2 to discover action rules reclassifying objects from class d0 to class d1 based on 6 classification rules {r1, r2,.., r6} extracted from some decision table.

5.1. Attributes a, b are stable.

5.2. Attribute a is stable

a b c d

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| r1 | 2 | 2 | 1 | 0 |
| r2 | 1 | 1 | 1 | 1 |
| r3 | 2 | 1 | 2 | 1 |
| r4 | 1 | 3 | 1 | 0 |
| r5 | 1 | 3 | 2 | 1 |
| r6 | 1 | 1 | 2 | 0 |

Table T.

**Solution (DEAR 2 algorithm).**

T(a1)

b c d

|  |  |  |  |
| --- | --- | --- | --- |
| r2 | 1 | 1 | 1 |
| r4 | 3 | 1 | 0 |
| r5 | 3 | 2 | 1 |
| r6 | 1 | 2 | 0 |

T(a1, d1)

b c

|  |  |  |
| --- | --- | --- |
| r2 | 1 | 1 |
| r5 | 3 | 2 |

T(a1,d0)

b c

|  |  |  |
| --- | --- | --- |
| r4 | 3 | 1 |
| r6 | 1 | 2 |

a1. c1.(b3 -> b1) => (d0 -> d1) Dom={x4}

a1.b3.(c1->c2) => (d0 -> d1) Dom={x4}

a1.b1.(c2 ->c1) => (d0 -> d1) Dom={x6}

a1. c2.(b1->b3) => (d0->d1) Dom={x6}

T(a2)

b c d

|  |  |  |  |
| --- | --- | --- | --- |
| r1 | 2 | 1 | 0 |
| r3 | 1 | 2 | 1 |

**Solution (DEAR 1 algorithm) –** first split by d, follow depth-first search

Homework – finish the solution

a b c d a,b -stable

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| r1 | 2 | 2 | 1 | 0 |
| r2 | 1 | 1 | 1 | 1 |
| r3 | 2 | 1 | 2 | 1 |
| r4 | 1 | 3 | 1 | 0 |
| r5 | 1 | 3 | 2 | 1 |
| r6 | 1 | 1 | 2 | 0 |

Table T.

T(d0)

a b c

|  |  |  |  |
| --- | --- | --- | --- |
| r1 | 2 | 2 | 1 |
| r4 | 1 | 3 | 1 |
| r6 | 1 | 1 | 2 |

T(d0,a1)

b c

|  |  |  |
| --- | --- | --- |
| r4 | 3 | 1 |
| r6 | 1 | 2 |

T(d0,a1,b3)

c

|  |  |
| --- | --- |
| r4 | 1 |

T(d0,a1,b1)

T(d0,a2)

b c

|  |  |  |
| --- | --- | --- |
| r1 | 2 | 1 |

T(d0,a2,b2)

c

|  |  |
| --- | --- |
| r1 | 1 |

T(d1)

a b c

|  |  |  |  |
| --- | --- | --- | --- |
| r2 | 1 | 1 | 1 |
| r3 | 2 | 1 | 2 |
| r5 | 1 | 3 | 2 |

T(d1,a1)

b c

|  |  |  |
| --- | --- | --- |
| r2 | 1 | 1 |
| r5 | 3 | 2 |

T(d1,a1,b1)

c

|  |  |
| --- | --- |
| r2 | 1 |

T(d1,a1,b3)

c

|  |  |
| --- | --- |
| r5 | 2 |

T(d0,a1,b3)

c

|  |  |
| --- | --- |
| r4 | 1 |

Action Rule constructed: a1.b3.(c1 → c2) → (d0 → d1)

T(d1,a2)

b c

|  |  |  |
| --- | --- | --- |
| r3 | 1 | 2 |

T(d1,a2,b1)

c

|  |  |
| --- | --- |
| r3 | 2 |

**Problem 5**

Let S=(X, {a, b, c, d}) be a decision system, where all attributes are flexible. Attribute d is the decision attribute. Find action rules reclassifying objects from the class d1 to d2 using action reducts.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a | b | c | d |
| x1 | a3 | b1 | c3 | d1 |
| x2 | a3 | b2 | c1 | d2 |
| x3 | a1 | b1 | c1 | d2 |
| x4 | a2 | b1 | c1 | d1 |
| x5 | a1 | b1 | c3 | d1 |
| x6 | a2 | b2 | c2 | d2 |

System S

Solution:

|  |  |  |  |
| --- | --- | --- | --- |
|  | X2 | X3 | X6 |
| X1 | b2, c1 | a1,c1 | a2,b2,c2 |
| X4 | a3, b2 | a1 | b2,c2 |
| X5 | a3, b2, c1 | c1 | a2,b2,c2 |

Reducts for objects in the desired class (x2, x3, x6):

R(x2)= (b2+c1)(a3+b2)(a3+b2+c1)= (b2+c1)(a3+b2)= b2.a3+b2+c1.a3+c1.b2= b2+c1.a3

R(x3)= (a1+c1)a1.c1= a1.c1

R(x6)= b2 + c2

Reducts: {c1,a3}, {a1,c1}, {b2}, {c2}

Action Rules Schema:

(c, → c1).(a, → a3) → (d, d1 → d2) Dom= {x1,x4,x5}

Finding rules for that schema

~~b1.(c,~~ → ~~c1).(a,~~ → ~~a3)~~ → ~~(d, d1~~ → ~~d2) Dom={x1,x4,x5}~~

~~b2.(c,~~ → ~~c1).(a,~~ → ~~a3)~~ → ~~(d, d1~~ → ~~d2) Dom=0~~

Action rules schema

(a, → a1).(c, → c1) → (d, d1 → d2) Dom={x1,x4,x5}

Finding rules for that schema

~~b1.(a,~~ → ~~a1).(c,~~ → ~~c1)~~ → ~~(d, d1~~ → ~~d2) Dom={x1,x4,x5}~~

~~b2.(a,~~ → ~~a1).(c,~~ → ~~c1)~~ → ~~(d, d1~~ → ~~d2) Dom=0~~

a1.(b, b1→ b2) → (d, d1 → d2) Dom={x5}

a2.(b, b1→ b2) → (d, d1 → d2) Dom= {x4}

a3.(b, b1→ b2) → (d, d1 → d2) Dom={x1}

c1.(b, b1→ b2) → (d, d1 → d2) Dom= {x4}

~~c2.(b, -> b2) => (d, d1 -> d2)~~

c3.(b, b1→ b2) → (d, d1 → d2) Dom={x1,x5}