**Problem 1.**

Find CNF for the following formula:

\((\forall x)(\exists y)(Q(x,y) \land \neg P(x,y)) \lor (\exists y)(\forall x)(Q(x,y) \land \neg R(x,y))\).

**Solution:**

1. Eliminate implication symbols.
2. Reduce scopes of negation symbols (negation symbol can be applied to at most one atomic formula).
3. Standardize variables.
4. Eliminate existential quantifiers.
5. Convert to prenex form (Skolemization).
6. Convert to conjunctive normal form.
7. Eliminate universal quantifiers.
8. Eliminate \land symbol.
9. Rename variables.

\((\forall x)(\exists y)(Q(x,y) \land \neg P(x,y)) \lor (\exists z)(\forall w)(Q(z,w) \land \neg R(z,w))\)

\((\forall x)(Q(x,f(x)) \land \neg P(x,f(x))) \lor (\forall z)(Q(z,A) \land \neg R(z,A))\)

\([Q(x,f(x)) \land \neg P(x,f(x))] \lor [Q(z,A) \land \neg R(z,A)]\)

\([Q(x,f(x)) \lor Q(z,A)] \land [Q(x,f(x)) \lor \neg R(z,A)] \land [\neg P(x,f(x)) \lor Q(z,A)] \land [\neg P(x,f(x)) \lor \neg R(z,A)].\)

**Problem 2.** In what order A*-algorithm will visit the nodes in the graph below (h(x) is the heuristic estimation of the distance from node x to one of the final nodes).
Problem 3. Formalize the following arguments and verify whether they are correct:

3.1) “If Carlo won the competition, then either Mario came second or Sergio came third. Mario didn’t come second. Thus, if Carlo won the competition, then Sergio didn’t come third.”

C - Carlo won the competition, M – Marion came second, S- Sergio came third.

3.2) “If you play and you study you’ll pass the exams, while if you play and don’t study you won’t pass. Thus, if you play, either you study and you’ll pass the exams, or you don’t study and you won’t pass.”

P- you play, S- you study, E- you pass the exam
Problem 4.

Use the truth tables method to determine whether \((\neg p \lor q) \land (q \to (\neg r \land \neg p)) \land (p \to r)\) is satisfiable.

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<tr>
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<th>(q \to (\neg r \land \neg p))</th>
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Problem 5. Follow RS method to check if \((a \to b) \to ((b \to c) \to (a \to c))\) is a tautology.

RS-Tree has to be constructed.

\[
(a \to b) \to ((b \to c) \to (a \to c))
\]

\[
\neg (a \to b), ((b \to c) \to (a \to c))
\]

\[
a, ((b \to c) \to (a \to c)); \neg b, ((b \to c) \to (a \to c))
\]

\[
a, \neg (b \to c), (a \to c)); \neg b, \neg (b \to c), (a \to c)
\]

\[
a, b, (a \to c); a, \neg c, (a \to c); \neg b, (a \to c); \neg b, c, (a \to c)
\]

\[
a, b, \neg a, c; a, \neg c, \neg a, c; \neg b, \neg a, c; \neg b, c, (a \to c)
\]

fundamental fundamental no

Problem 6. Consider the following sentences: 1. All actors and journalists invited to the party are late. 2. There is at least a person who is on time. 3. There is at least an invited person who is neither a journalist nor an actor.

Formalize the sentences and prove that 3. is not a logical consequence of 1 and 2.

Predicates: \(I(x)\) – x invited to party, \(L(x)\) – x is late, \(A(x)\) – x is actor, \(J(x)\) – x is journalist

\(I(x) \land (A(x) \lor J(x)) \to L(x)\) is equivalent to: \(\neg L(x) \to \neg I(x) \lor \neg (A(x) \lor J(x))\)

We know that \((\exists x) \neg L(x)\) it means that \(\neg L(P)\) is true for some person P and the same \(\neg I(P) \lor \neg (A(P) \lor J(P))\) is true.

It shows that non invited journalists/actors can be on time.
**Problem 7.** Convert the following formulas to CNF and check if they are satisfiable:

1. \( \neg(((a \rightarrow b)) \rightarrow a) \rightarrow a \)
2. \( \neg((p \rightarrow (q \rightarrow r))) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \)
3. \( \neg((\forall x)(\exists y)[p(x) \rightarrow \neg q(x,y)] \land (\exists x)(\forall y)q(x,y) \)
4. \( (\exists x)(\forall y)[p \land \neg r(x,y) \land s(x)] \rightarrow (\neg q \land s(x)) \)

**Solution:**

Number (3)

\[ \neg((\forall x)(\exists y)[p(x) \lor \neg q(x,y)] \land (\exists w)(\forall z)q(w,z) \]

\[ (\exists x)(\forall y)[\neg p(x) \land q(x,y)] \land (\exists w)(\forall z)q(w,z) \]

\[ (\forall y)[\neg p(B) \land q(B,y)] \land (\forall z)q(A,z) \]

\[ \neg p(B) \land q(B,y) \land q(A,z) \]

**Problem 8.** Consider the following facts:

Anyone whom Mary loves is a football star. Any student who does not pass does not play. John is a student. Any student who does not study does not pass. Anyone who does not play is not a football star.

**Predicates:** 
- \( M(x) \) – [Mary loves \( x \)]
- \( F(x) \) – [\( x \) is a football star]
- \( P(x) \) – [\( x \) passed the exam]
- \( G(x) \) – [\( x \) play football]
- \( S(x) \) – [\( x \) is a student]
- \( St(x) \) – [\( x \) studies]

Prove that: If John does not study, then Mary does not love John.

Formulas:

\( M(x) \rightarrow F(x) \)

\[ [S(x) \land \neg P(x)] \rightarrow \neg G(x) \]

\( S(J) \)

\[ [S(x) \land \neg St(x)] \rightarrow \neg P(x) \]

\[ \neg G(x) \rightarrow \neg F(x) \]

Prove that: \( \neg St(J) \rightarrow \neg M(J) \)

After conversion to the set of clauses, we get:

1. \( \neg M(x) \lor F(x) \)
2. \( \neg S(x) \lor P(x) \lor \neg G(x) \)
3. \( S(J) \)
Problem 9. Consider the following axioms:

Every coyote chases some roadrunner. Every roadrunner who says \textasciitilde{``beep-beep''} is smart. No coyote catches any smart roadrunner. Any coyote who chases some roadrunner but does not catch it is frustrated.

Prove that: If all roadrunners say \textasciitilde{``beep-beep''}, then all coyotes are frustrated.

Predicates: \textup{Ch}(x,y) – [x chases y], \textup{Cat}(x,y) – [x catches y], \textup{S}(x) – [x says \textasciitilde{``beep-beep''}], \textup{Sm}(x) – [x is smart], \textup{F}(x) – [x is frustrated].

Functors: \textup{Co}(x) – [x is coyote], \textup{R}(x) – [x is roadrunner]

Formulas:

\((\forall x)(\exists y)\textup{Ch}(\textup{Co}(x), \textup{R}(y)), \ \textup{S}(\textup{R}(y)) \rightarrow \textup{Sm}(\textup{R}(y)), \ \neg(\exists x)\left[\textup{Cat}(\textup{Co}(x), \textup{R}(y)) \land \textup{Sm}(\textup{R}(y))\right],\)

\([\textup{Ch}(\textup{Co}(x), \textup{R}(y)) \land \neg\textup{cat}(\textup{Co}(x)), \textup{R}(y))] \rightarrow \textup{F}(\textup{Co}(x))\)

Prove that: \((\forall y)\textup{S}(\textup{R}(y)) \rightarrow (\forall x)\textup{F}(\textup{Co}(x))\)

Negation of the goal: \(\neg[(\forall y)\textup{S}(\textup{R}(y)) \rightarrow (\forall x)\textup{F}(\textup{Co}(x))]\) which is \(\textup{S}(\textup{R}(y)) \land \neg\textup{F}(\textup{Co}(fo))\)

After conversion to the set of clauses, we get:

\((1) \textup{Ch}(\textup{Co}(x), \textup{R}(\textup{f1}(x)))) \quad (2) \neg\textup{S}(\textup{R}(y)) \lor \textup{Sm}(\textup{R}(y)) \quad (3) \neg\textup{Cat}(\textup{Co}(x), \textup{R}(y)) \lor \neg\textup{Sm}(\textup{R}(y))\)

\((5) \neg\textup{Ch}(\textup{Co}(x), \textup{R}(y)) \lor \textup{cat}(\textup{Co}(x), \textup{R}(y)) \lor \textup{F}(\textup{Co}(x))\)

Prove that: \(\neg(\forall y)\textup{S}(\textup{R}(y)) \lor (\forall x)\textup{F}(\textup{Co}(x))\)

Its negation: \((6) \textup{S}(\textup{R}(y)) \quad (7) \neg\textup{F}(\textup{Co}(fo))\)

Resolution steps: \((1)+(5)+(7)\) gives \((8): \textup{cat}(\textup{Co}(fo), \textup{R}(\textup{f1}(x)))\)
Problem 10. Consider the following axioms: Every child loves every candy. Anyone who loves some candy is not a nutrition fanatic. Anyone who eats any pumpkin is a nutrition fanatic. Anyone who buys any pumpkin either carves it or eats it. John buys a pumpkin. Lifesavers is a candy.

Prove that: If John is a child, then John carves some pumpkin.

Functors:
\( ca(y) \) – \( y \) is candy; \( p(y) \) – \( y \) is pumpkin; \( J \) – John; \( Life \) – Lifesavers

Predicates:
\( nf(x) \) – \( x \) is nutrition fanatic; \( L(x,y) \) – \( x \) loves \( y \); \( Et(x,y) \) – \( x \) eats \( y \);
\( Car(x,y) \) – \( x \) carves \( y \); \( B(x,y) \) – \( x \) buys \( y \); \( C(x) \) – \( x \) is a child

Formulas:
\( C(x) \rightarrow L(x, ca(y)) \)
\( (\exists y) L(x, ca(y)) \rightarrow \neg nf(x) \)
\( Et(x,p(y)) \rightarrow nf(x) \)
\( B(x, p(y)) \rightarrow Et(x,p(y)) \vee Car(x, p(y)) \)
\( B(J, p(y)) \)

Prove that: \( C(J) \rightarrow (\exists y)Car(J,p(y)) \)

After conversion to the set of clauses, we get:
\( 1 \) \( \neg C(x) \vee L(x, ca(y)) \)
\( 2 \) \( \neg C(x) \vee Et(x,p(y)) \vee \neg nf(x) \)
\( 3 \) \( \neg Et(x,p(y)) \vee nf(x) \)
\( 4 \) \( \neg B(x, p(y)) \vee Et(x,p(y)) \vee Car(x, p(y)) \)
\( 5 \) \( B(J, p(y)) \)

Negation of the goal: \( \neg [\neg C(J) \vee (\exists y)Car(J,p(y))] \) which is:
\( 7 \) \( C(J) \)
\( 8 \) \( \neg Car(J,p(y)) \)

Resolution steps:
\( 1+7 \) gives (9): \( L(J, ca(y)) \)
\( 9+2 \) gives (10): \( \neg nf(J) \)
\( 10+3 \) gives (11): \( \neg Et(J,p(y)) \)
\( 4+11+5+8 = NIL \)
Problem 11. Consider the following axioms: Every child sees some witch. No witch has both a black cat and a painted hat. Every witch is good or bad. Every child who sees any good witch gets candy. Every witch that is bad has a black cat. Prove that: If every witch that is seen by any child has a painted hat, then every child gets candy.

Functors:
Ch(x) – x is a child;  W(y) – y is a witch;  bc – black cat;  ph – painted hat;  can – candy

Predicates:
H(y,z) – y has z;  G(y) - y is good;  B(y) – y is bad;  S(x,y) – x sees y;  Get(x,y) – x gets y.

Formulas:
\((\forall x)(\exists y)S(Ch(x),W(y))\)
\(\neg((\exists y)[H(W(y),bc) \land H(W(y), ph)])\)
\(G(W(y)) \lor B(W(y))\)
\(S(Ch(x), W(y)) \land G(W(y)) \rightarrow Get(Ch(x), can)\)
\(B(W(y)) \rightarrow H(W(y), bc)\)

Prove that: \(S(Ch(x), W(y)) \land H(W(y), ph) \rightarrow (\forall w)Get(Ch(w), can)\)

After conversion to the set of clauses, we get:

1. \(S(Ch(x), W(f(x)))\)
2. \(\neg H(W(y), bc) \lor \neg H(W(y), ph)\)
3. \(G(W(y)) \lor B(W(y))\)
4. \(\neg S(Ch(x), W(y)) \lor \neg G(W(y)) \lor Get(Ch(x), can)\)
5. \(\neg B(W(y)) \lor H(W(y), bc)\)

Prove that: \(\neg S(Ch(x), W(y)) \lor \neg H(W(y), ph) \lor Get(Ch(w), can)\)

Negation of the goal: (6) \(S(Ch(x), W(y))\), (7) \(H(W(y), ph)\), (8) \(\neg Get(Ch(w), can)\)

Resolution steps:
(7)+(2) gets (9): \(\neg H(W(y), bc)\)
(9)+(5) gets (10): \(\neg B(W(y))\)
(10)+(3) gets (11): \(G(W(y))\)
(4)+(1)+(11)+(8) = NIL

Problem 12. Consider the following axioms: Every boy or girl is a child. Every child gets a doll or a train or a lump of coal. No boy gets any doll. No child who is good gets any lump of coal. Prove that: If no child gets a train, then no boy is good.

Predicates:  B(x) – x is a boy,  G(x) – x is a girl,  C(x) – x is a child,  Gets(x,y) – x gets y,
Good(x) – x is good.

Constants:  D - Doll, T - Train, LC – lump of coal,
Facts:

\[
[B(x) \lor G(x)] \rightarrow C(x), \quad C(x) \rightarrow [\text{Gets}(x, D) \lor \text{Gets}(x, T) \lor \text{Gets}(x, LC)], \quad B(x) \rightarrow \neg \text{Gets}(x, D),
\]

\[
[C(x) \land \text{Good}(x)] \rightarrow \neg \text{Gets}(x, LC)
\]

Prove that: \([C(x) \rightarrow \neg \text{Gets}(x, T)] \rightarrow [B(x) \rightarrow \neg \text{Good}(x)]\)

Negation of the goal: \(\neg [[C(x) \rightarrow \neg \text{Gets}(x, T)] \rightarrow [B(x) \rightarrow \neg \text{Good}(x)]]\)

\(\neg [\neg [C(x) \lor \neg \text{Gets}(x, T)] \lor \neg B(x) \lor \neg \text{Good}(x)]\)

\(\neg C(x) \lor \neg \text{Gets}(x, T) \land \neg [B(x) \land \neg \text{Good}(x)]\)

So, we get:

(1) \(\neg B(x) \lor C(x)\)

(2) \(\neg G(x) \lor C(x)\)

(3) \(\neg C(x) \lor \text{Gets}(x, D) \lor \text{Gets}(x, T) \lor \text{Gets}(x, LC)\)

(4) \(\neg B(x) \lor \neg \text{Gets}(x, D)\)

(5) \(\neg C(x) \lor \neg \text{Good}(x) \lor \neg \text{Gets}(x, LC)\)

(6) \(\neg C(x) \lor \neg \text{Gets}(x, T)\)

(7) \(B(x)\)

(8) \(\text{Good}(x)\)

(8)+(1): (9) \(C(x)\)

(5)+(8)+(9): (10) \(\neg \text{Gets}(x, LC)\)

(9)+(6): (11) \(\neg \text{Gets}(x, T)\)

(4)+(7): (12) \(\neg \text{Gets}(x, D)\)

Negation of the goal:

\(\neg [[C(x) \land \text{Young}(x)] \lor [\text{Hr}(x) \lor \text{H(x)}] \rightarrow [C(x) \rightarrow \text{Hp}(x)]]\)

**Problem 13.** Consider the following axioms: Every child who finds some [thing that is an] egg or chocolate bunny is happy. Every child who is helped, finds some egg. Every child who is not young or who tries hard, finds some chocolate bunny. Prove that: If every young child tries hard or is helped, then every child is happy

**Solution:**

**Constants** (functor of order zero): \(\text{Eg} – \text{egg}; \quad \text{Ch} – \text{chocolate bunny}\)

**Predicates:** \(C(x) – x \text{ is a child}; \quad F(x,y) – x \text{ finds } y; \quad H(x) – x \text{ is helped}; \quad Y(x) – x \text{ is young};\)

\(\text{Hp}(x) – x \text{ is happy}; \quad \text{Hr}(x) – x \text{ tries hard}\)

**Formulas:**

\(C(x) \land [F(x,\text{Eg}) \lor F(x,\text{Ch})] \rightarrow \text{Hp}(x)\)

\(C(x) \land H(x) \rightarrow F(x,\text{Eq})\)

\(C(x) \land [\neg Y(x) \lor \text{Hr}(x)] \rightarrow F(x,\text{Ch})\)

**Prove that:** \([[C(x) \land Y(x)] \rightarrow [\text{Hr}(x) \lor H(x)] \rightarrow [C(x) \rightarrow \text{Hp}(x)]]\)

After conversion to clausal form we get:

(1) \(\neg C(x) \lor \neg F(x,\text{Eq}) \lor \text{Hp}(x)\)

(2) \(\neg C(x) \lor \neg F(x,\text{Ch}) \lor \text{Hp}(x)\)

(3) \(\neg C(x) \lor \neg H(x) \lor F(x,\text{Eq})\)

(4) \(\neg C(x) \lor Y(x) \lor F(x,\text{Ch})\)

(5) \(\neg C(x) \lor \neg \text{Hr}(x) \lor F(x,\text{Ch})\)

**Negation of the goal:** \(\neg [[[\neg C(x) \land Y(x)] \lor [\text{Hr}(x) \lor H(x)] \rightarrow [C(x) \rightarrow \text{Hp}(x)]]\)
\[\neg \left( \neg [C(x) \land Y(x)] \lor \neg [Hr(x) \lor H(x)] \lor \neg C(x) \lor Hp(x) \right) =
\]
\[\left[ C(x) \land Y(x) \right] \land \left[ \left[ Hr(x) \lor H(x) \right] \land \left[ C(x) \land \neg Hp(x) \right] \right] \]

(6) \( C(x) \)  (7) \( Y(x) \)  (8) \( Hr(x) \lor H(x) \)  (9) \( \neg Hp(x) \)

(1)+(6)+(9): (10) \( \neg F(x, \text{Eq}) \)

(10)+(3)+(6): (11) \( \neg H(x) \)  ;  (11)+(8): (12) \( Hr(x) \)

(5)+(6)+(12): (13) \( F(x, \text{Ch}) \)  ;  (2)+(6)+(13)+(9) = \text{NIL}