Predicate Calculus

Alphabet:
Constants: a, b, c, d
Variables: x, y, w, z
Functors: f, g, h
Predicates: p, q, r
Connectives: $\vee$, $\wedge$, $\Rightarrow$, $\neg$
Quantifiers: $\forall$, $\exists$

Terms:
The least set satisfying the properties:

1) Constants and variables are terms.
2) If $t_1, t_2, \ldots, t_k$ are terms and f is k-argument functor, then $f(t_1, t_2, \ldots, t_k)$ is a term.

Example: *(+(x,2),y) is a term. It can be also written as ((x+2)*y).

Formulas:
The least set satisfying the properties:

1. $t_1, t_2, \ldots, t_k$ are terms and p is k-argument predicate, then $p(t_1, t_2, \ldots, t_k)$ is a formula (called atomic formulas).
2. if α, β are formulas, then $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \Rightarrow \beta)$, $\neg\alpha$ are formulas.
3. if $\alpha(x)$ is a formula, then $(\forall x)\alpha(x)$, $(\exists x)\alpha(x)$ are formulas.

Control Strategies for Resolution Methods

Resolution is an important rule of inference that can be applied to a certain class of well formed formulas (wffs) called clauses. A clause is defined as a wff consisting of a disjunction of literals. Literal is defined as atomic formula or its negation. The resolution process, when it is applicable, is applied to a pair of parent clauses to produce a derived clause.

Process of converting any predicate calculus wff to a set of clauses:

1) Eliminate implication symbols.
2) Reduce scopes of negation symbols (negation symbol can be applied to at most one atomic formula)
3) Standardize variables
4) Eliminate existential quantifiers
5) Convert to prenex form (Skolemization)
6) Convert to conjunctive normal form
7) Eliminate universal quantifiers
8) Eliminate $\wedge$ symbol
9) Rename variables

Example: $(\forall x)[P(x) \Rightarrow ((\forall y)(P(y) \Rightarrow P(f(x,y)))] \wedge (\forall y)[Q(x,y) \Rightarrow P(y)])]$.

Breadth-First Strategy
All of the first-level resolvents are computed first, then the second-level resolvents, and so on. A first-level resolvent is one between two clauses in the base set; an i-th level resolvent is one whose deepest parent is an (i-1)-th level resolvent. The breadth-first strategy is complete, but it is grossly inefficient.

Example: Find all resolvents for the following base set: $\neg I(z) \vee R(z)$, $I(a)$, $\neg R(x) \vee L(x)$, $\neg D(y) \vee \neg L(y)$, $D(a)$

The Set-of-Support Strategy
At least one parent of each resolvent is selected from among the clauses resulting from the negation of the goal or from their descendants. The strategy is complete.

The Unit-Preference Strategy
It is a modification of the set-of support strategy in which, instead of filling out each level in breadth-first fashion, we try to select a single-literal clause (called a unit) to be a parent in a resolution. Every time units are used in resolution, the resolvent have fewer literals than do their other parents. The strategy is complete.

The Linear-Input Form Strategy
Each resolvent has at least one parent belonging to the base set. The strategy is not complete.

The Ancestry-Filtered Form Strategy
Each resolvent has a parent that is either in the base set or that is an ancestor of the other parent. The strategy is complete.

Example: Find all resolvents for the following base set: $\neg Q(x) \vee \neg P(x)$, $\neg Q(y) \vee \neg P(y)$, $\neg Q(w) \vee P(w)$, $Q(u) \vee P(a)$.

Problem 1.
Translate into symbols the following statements, using quantifiers, variables and predicate symbols.
If some trains are late then all trains are late.
Everyone is loyal to someone
Some people hate everyone.
No mouse is heavier than any elephant.

Problem 2.
Consider the following statements:
- *if the maid stole the jewelry, then the butler wasn’t guilty.*
- *either the maid stole the jewelry or she milked the cows.*
- *if the maid milked the cows, then the butler got his cream.*
- *therefore, if the butler was guity, then he got his cream.*

a) Express these statements in the propositional calculus.

b) Express the negation of the conclusion in clause form.
c) Demonstrate that the conclusion is valid, using resolution in the propositional calculus.

Problem 3.
Check if the formula $(A \to \sim A) \to (A \wedge (\sim A \to A))$ is a tautology.

Problem 4.
Find CNF for the following formula:
$(\forall x)(\exists y)[Q(x,y) \wedge \neg P(x,y)] \vee (\exists y)(\forall x)[Q(x,y) \wedge \neg R(x,y)]$.

Problem 5.
Translate into symbols the following statements, using quantifiers, variables and predicate symbols:
- *Tony, Mike, and John belong to the Alpine club.*
- *Every member of the Alpine club who is not a skier is a mountain climber.*
- *Mountain climbers do not like rain and anyone who does not like snow is not a skier.*
- *Mike dislikes whatever Tony likes and likes whatever Tony dislikes.*
- *Tony likes rain and snow.*

Use resolution to show that:
- *There a member of the Alpine club who is a mountain climber but not a skier*

Problem 6.
Translate into symbols the following statements:
- *If a course is easy, some students are happy.*
- *If a course has a final, no students are happy.*

Use resolution to show that: *If a course has a final, the course is not easy.*