Dijkstra’s Algorithm [shortest path]

Example
An example of an A* algorithm in action where nodes are cities connected with roads and h(x) is the straight-line distance to target point:

A* - Algorithm

\[ f(n) = g(n) + h(n) \] /expand node with minimum f(n)/

\[ g(n) = \text{cost to get to } n \text{ from the root} \]
\[ h(n) = \text{cost to get to goal from } n \text{ (optimistic function)} \]

\[ f(n) \text{ never overestimates cost of a solution through } n \text{ /it is admissible/} \]
\[ h(n) \leq c(n, a, n') + h(n') \text{ is monotonic /f(n) is not decreasing along any path/} \]

Remark: A* is simplified to Dijkstra’s algorithm if h(n)=0
**f(n) – admissible & monotonic -> consistent**

![Diagram showing the admissibility and monotonicity of f(n)]

**Example (8-puzzle problem)**

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Goal State</th>
<th>Hamming</th>
<th>Manhattan</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 1 3</td>
<td>1 2 3 4 5 6 7 8</td>
<td>5 + 0</td>
<td>10 + 0</td>
</tr>
<tr>
<td>4 2</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 6 5</td>
<td>1 1 0 0 1 1 0 1</td>
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**Option I (Hamming Distance)**

\[ c(x) = f(x) + h(x) \]

- \( f(x) \) is the length of the path from root to \( x \) (the number of moves so far) and
- \( h(x) \) is the number of non-blank tiles not in their goal position (the number of mis-placed tiles). There are at least \( h(x) \) moves to transform state \( x \) to a goal state

**Option II (Manhattan Distance – better approximation)**
The sum of the Manhattan distances (sum of the vertical and horizontal distance) from the blocks to their goal positions.

Option I & Option II – both admissible (do not overestimate the cost)

**Some 8-puzzles are not solvable**

![8-puzzles](image)

**Interesting property**

For every digit count the number of smaller digits which follow that digit.

![Board counting](image)

**Fact:** If this number is odd, then the puzzle is solvable with goal state given below.

<p>| | | |</p>
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**Game 2:** [Cannibals & Missionaries](#)
f(n)=g(n) + h(n) ; 2 traveling from right to left and 1 traveling from left to right

h(n) = 2 * (m(R) + c(R)), where

m(R) - number of missionaries on the right site of the river

c(R) – number of cannibals on the right site of the river

f(n) – admissible (can we find better approximation?)

**New Problem:** Farmer went to a market and purchased a fox, a goose, and a bag of beans. On his way home, he came to the bank of a river and rented a boat. But crossing the river by boat, the farmer could carry only himself and a single one of his purchases: the fox, the goose, or the bag of beans. If left unattended together, the fox would eat the goose, or the goose would eat the beans.
Eight Puzzle

A, \( h(A) = 10 \)
B, \( h(B) = 11 \)
C, \( h(C) = 9 \)
D, \( h(D) = 9 \)
E, \( h(E) = 11 \)

\[ f(n) = g(n) + h(n) \]

Goal State

H – Manhattan Distance