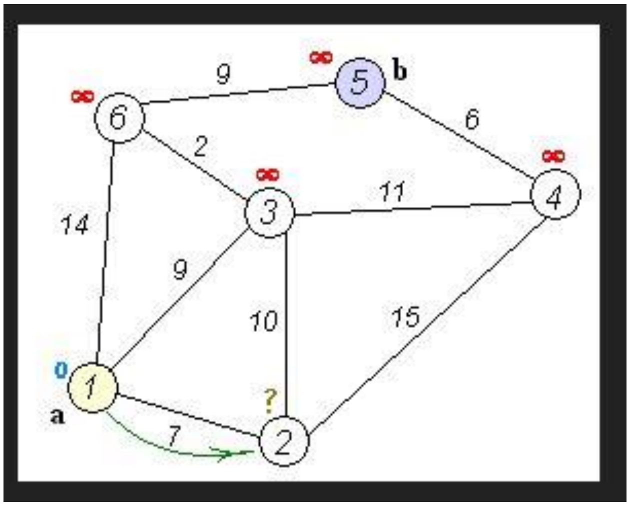
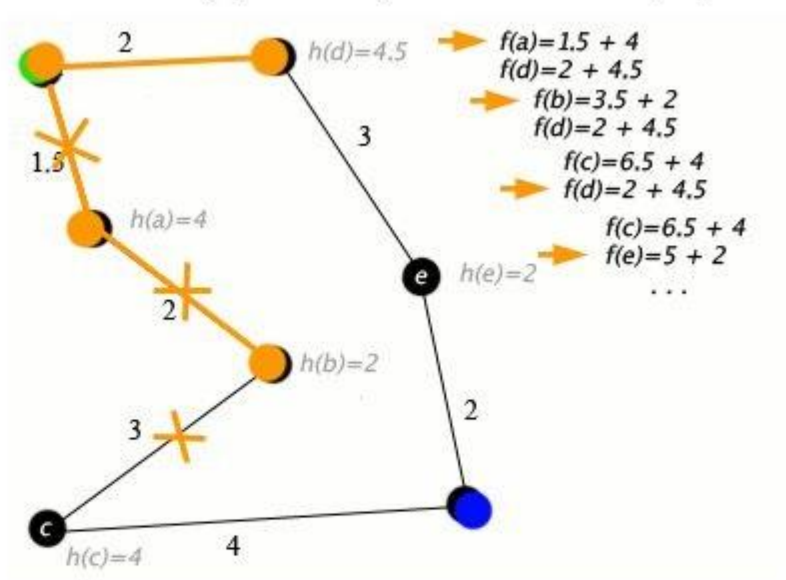


Dijkstra’s Algorithm [shortest path]

Example

An example of an A* algorithm in action where nodes are cities connected with roads and h(x) is the straight-line distance to target point:



A* - Algorithm

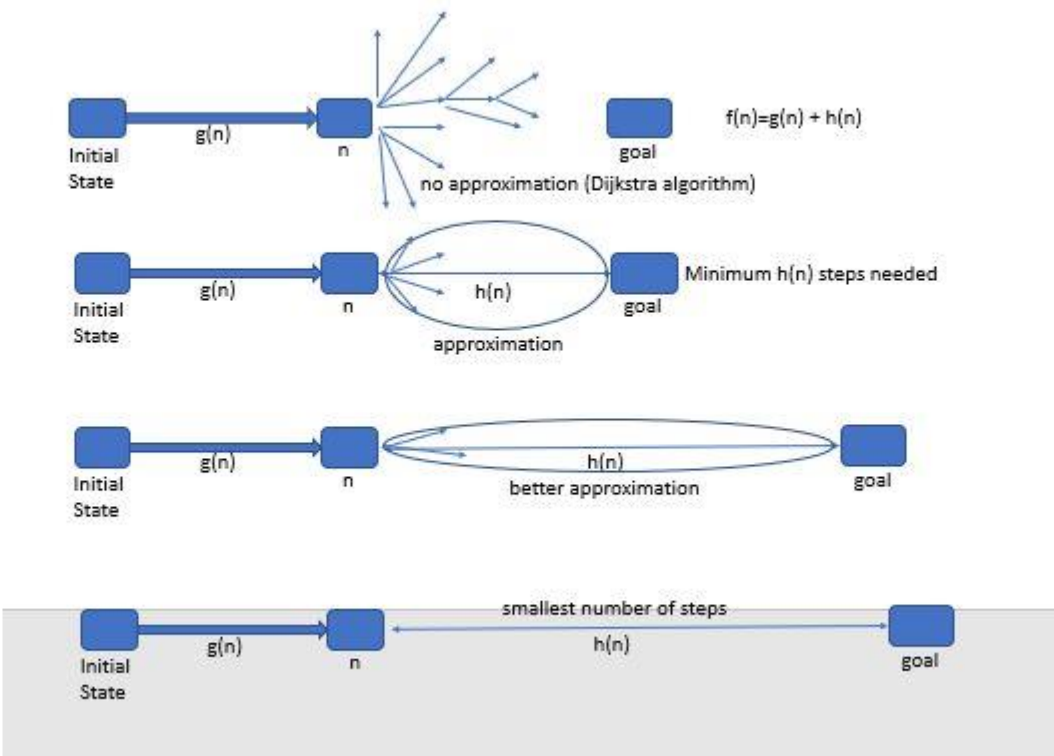
f(n) = g(n) + h(n) /expand node with minimum f(n)/

g(n) = cost to get to n from the root, **h(n)** = cost to get to goal from n (optimistic function)

f(n) never overestimates cost of a solution through n /it is **admissible**/

$h(n) \leq c(n, a, n') + h(n')$ is **monotonic** /f(n) is not decreasing along any path/

Remark: A* is simplified to Dijkstra’s algorithm if h(n)=0 f(n) – **admissible & monotonic -> consistent**



Example (8-puzzle problem)

```
8  1  3       1  2  3     1  2  3  4  5  6  7  8     1  2  3  4  5  6  7  8
4     2       4  5  6     ----------------------     ----------------------
7  6  5       7  8        1  1  0  0  1  1  0  1     1  2  0  0  2  2  0  3

initial        goal          Hamming = 5 + 0           Manhattan = 10 + 0
```

Option I (Hamming Distance)

```
c(x) = f(x) + h(x) where
   f(x) is the length of the path from root to x
        (the number of moves so far) and
   h(x) is the number of non-blank tiles not in
        their goal position (the number of mis-
        -placed tiles). There are at least h(x)
        moves to transform state x to a goal state
```

Option II (Manhattan Distance – better approximation)

The sum of the Manhattan distances (sum of the vertical and horizontal distance) from the blocks to their goal positions.

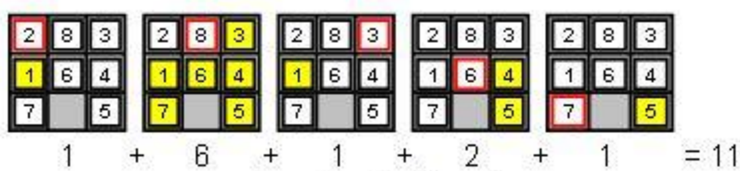
Option I & Option II – both admissible (do not overestimate the cost)

Some 8-puzzles are not solvable

```
 1  2  3          1  2  3  4
 4  5  6          5  6  7  8
 8  7             9 10 11 12
                 13 15 14
unsolvable
                 unsolvable
```

Interesting property

For every digit count the number of smaller digits which follow that digit.

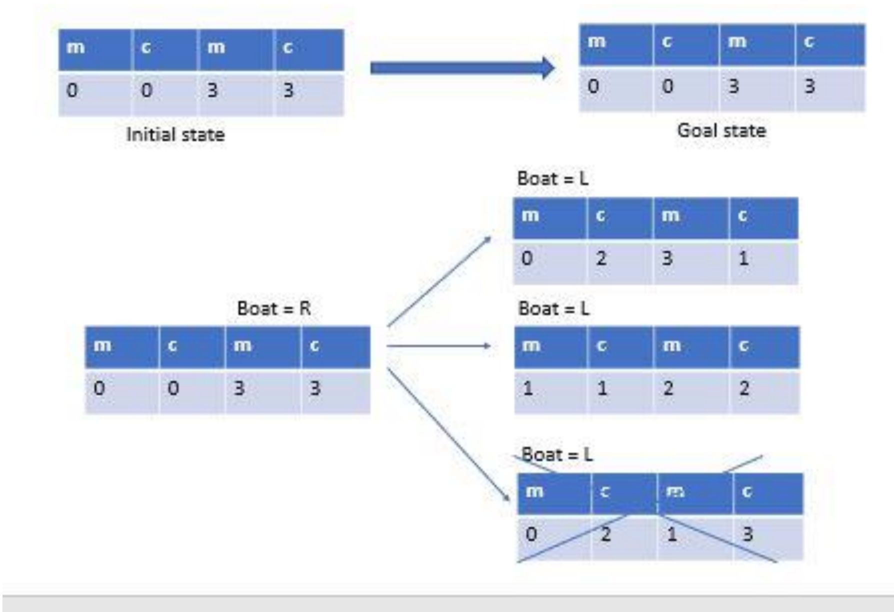


Counting the Board

Fact: If this number is odd, then the puzzle is solvable with goal state given below.

1	2	3
8		4
7	6	5

Game 2: Cannibals & Missionaries 



f(n)=g(n) + h(n) ;   2 traveling from right to left and 1 traveling from left to right

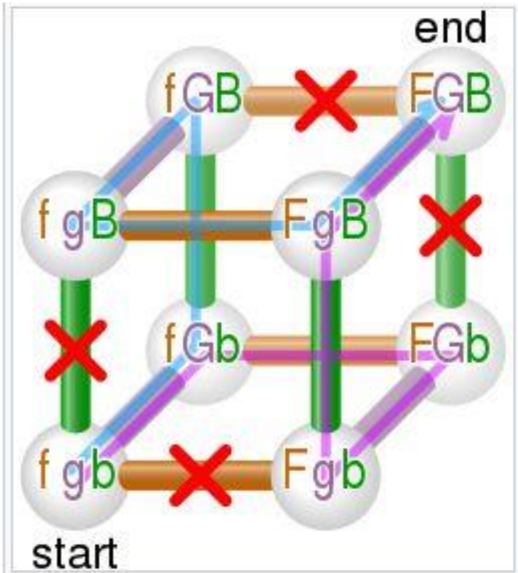
h(n) = 2 * (m(R) + c(R)), where

m(R) - number of missionaries on the right site of the river

c(R) – number of cannibals on the right site of the river

f(n) – admissible (can we find better approximation?)

New Problem: Farmer went to a market and purchased a fox, a goose, and a bag of beans. On his way home, he came to the bank of a river and rented a boat. But crossing the river by boat, the farmer could carry only himself and a single one of his purchases: the fox, the goose, or the bag of beans. If left unattended together, the fox would eat the goose, or the goose would eat the beans.



If 0 items moved, then h(n)=5. If G moved, then h(n)=4. If 2 FG or GB moved, then h(n)=3, ……..