Lecture 4

Uninformed Searches (cont)
Cost of Breadth-first Search (BFS)

- $b$ – max branching factor (infinite?)
- $d$ – depth to shallowest goal node
- $m$ – max length of any path (infinite?)

- Analysis of space and time performance….
  - $g(n)$ is $O(f(n))$ means that there are positive constants $c$ and $n_0$ such that:
    $0 \leq g(n) \leq cf(n)$ for all $n \geq n_0$
Cost of Depth-first Search

- time complexity = $O(b^m)$
  - length of longest path anchors upper bound
- space complexity = $O(bm)$

- What’s an example that highlights difference in performance between BFS and DFS?
Iterative Deepening

*Essentially DFS with a depth limit*

**Why?**

- Remember the space complexity, $O(bm)$, and time complexity, $O(b^m)$, of DFS?
- So... limit $m$ to be small
- But what if $m$ is smaller than $d$?
  - we’ll never find solution
- So... increment depth limit starting from 0
Bidirectional Search

Search from goal to start

Search from start to goal

Link — Procedure

Graph
Bidirectional Search

- Do you always know the predecessors of a goal state?
- Do you always know the goal state?
  - 8-puzzle [Link]
  - Puzzle Game - [Cannibals & Missionaries][Link]
  - Chess
Avoid Repeated States

How might you create repeated states in 8-puzzle?

How can you detect repeated states?

What about preserving lowest cost path among repeated states?

- uniform-cost search (shortest path) and
- BFS w/ constant step costs
Interesting problems

**Exercise 3.9:**

- 3 cannibals and 3 missionaries and a boat that can hold one or two people are on one side of the river. Get everyone across the river.

- 8-puzzle and 15-puzzle, invented by Sam Loyd in 1870s. Think about search space.

- Rubik’s cube [Link](#)

- Traveling Salesman Problem (TSP) [Link](#)
Farmer, Fox, Goose, and Bag of Corn

**Puzzle concerning a farmer, a fox, a goose, and a bag of corn.**

The farmer wants to get all of this stuff across a river. His boat will only hold himself, and one of the other three items. The fox will eat the goose if the farmer is not present, and the goose will eat the corn if the farmer is not present. How can the farmer transport all three items across the river?
Chapter 4 – Informed Search

INFORMED?

• Uses problem-specific knowledge beyond the definition of the problem itself
  – selecting best lane in traffic
Best-first Search

**Use an evaluation function to select node to expand**

- $f(n) =$ evaluation function = expected distance to goal
- select the node that minimizes $f(n)$
- but if we knew the ‘best’ node to explore it wouldn’t be search!!!
- Let’s use heuristics for our evaluation functions
Heuristics

A function, $h(n)$, that estimates cost of cheapest path from node $n$ to the goal

- $h(n) = 0$ if $n =$ goal node
A* (A-star) Search

**Combine two costs**

- \( f(n) = g(n) + h(n) \)
  - \( g(n) \) = cost to get to \( n \)
  - \( h(n) \) = cost to get to goal from \( n \)

**Minimize \( f(n) \)**
A* is Optimal?

A* can be optimal if $h(n)$ satisfies conditions

- $h(n)$ never overestimates cost to reach the goal
  - *admissible heuristic*
  - $h(n)$ is optimistic
  - $f(n)$ never overestimates cost of a solution through $n$

- Proof of optimality?
A* is Optimal

We must prove that A* will not return a suboptimal goal or a suboptimal path to a goal

• Let G be a suboptimal goal node
  – f(G) = g(G) + h(G)
  – h(G) = 0 because G is a goal node
  – f(G) = g(G) > C* (because G is suboptimal)
A* is Optimal

We must prove that A* will not return a suboptimal goal or a suboptimal path to a goal.

- Let G be a suboptimal goal node:
  - $f(G) = g(G) + h(G)$
  - $h(G) = 0$ because G is a goal node.
  - $f(G) = g(G) > C^*$ (because G is suboptimal).

- Let n be a node on the optimal path:
  - because $h(n)$ does not overestimate
  - $f(n) = g(n) + h(n) \leq C^*$

- Therefore $f(n) \leq C^* < f(G)$
  - node n will be selected before node G